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Sports Training Method Based on Multiobjective Optimization

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Abstract

At present, most of the sports training methods adopt the recombination operator that is designed for single objective optimization. Through the validation or experimental, several typical single objective sports training methods are analyzed and proven that they are not applicable for some multiobjective optimization problems. The multiobjective optimization sports training method based on the mixture Kalman model (Multiobjective evolutionary algorithm -based on decomposition and mixture Kalman models, MOPE for short) is proposed. This algorithm firstly applies an improved mixture Kalman model to carry out sampling to the group sports training and generate new individuals, and then makes use the greedy strategy to update the group. In view of the multiobjective optimization problem that is of complicated Pareto Front, the test results show that for the majority of the given athletes, this training method can achieve good effect.

Keywords: *Multiobjective Optimization, Evolutionary Algorithms, MOBC, Mixture Kalman Probability Model;*

1. Introduction

Multiobjective optimization problem (Referred to as "MOP" for short) is a type of challenging optimization problems widely existing in the field of scientific research and production application. The conflicts between the MOP objectives can cause the result that there is usually not an optimal solution that can meet all of the optimization objectives. As a result, we often need to make a compromise on each objective Pareto optimal solution set [1]. For the general MOP problem, there is no analytical method yet. Therefore, we often make use of the computer algorithms to obtain an approximation of Pareto solution set. Evolutionary algorithm (Referred to as "EA" for short) is a kind of imitation of nature, especially the biological evolution process to solve the complicated optimization problems of computer algorithm model. EA algorithm has two important features: (1) For the nature of solving problems, such as continuity and differentiability, no special assumptions are made; (2) Based on population search, namely, the adoption of multi-point search problem optimal solution at the same time. Therefore, EA algorithm is especially applicable for solving complicated nonlinear and the black box MOP problem.

Since 1985, the first kind of sports training methods (Multiobjective evolutionary algorithm, referred to as the “MOEA” for short) was put forward [2], MOEA has become one of the mainstream approaches in the problem solving of MOP; At the same time, MOEA has also become one of the most popular research

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direction in the field of EA [3-5]. Internationally, NSGA- II[6], SPEA2[7], PAES[8], IBEA[9,10], MOBC[11 and MOBC [11], as the representatives of MOEA algorithm have been widely used in many application fields. Meanwhile, MOEA research in China has gained rapid development [12-14], the research focuses in algorithm design, such as the multiobjective algorithm based on design of experiment [15-17], multiple objectives based on differential evolution algorithm [17,18], as well as the multiobjective optimization algorithm based on particle swarm [19,20]. On the foundation of the above research foundation, the literature [12] proposed that: In the frame of the MOBC algorithm, the recombination operator based on Kalman probability model to sports training in population and sampling. The preliminary experiment results show that the Kalman probability model can effectively approximate Pareto optimal solution set of the manifold, MOP problem with complicated geometry to obtain ideal experiment result. In this paper, the algorithm made further improvement, puts forward a new model based on decomposition and mixture Kalman multiobjective optimization of sports training methods (MOBC - GM).

In this paper, it mainly puts forward the improved recombination operator based on the mixture Kalman probability model, the operator makes full use of the MOBC neighbor individual similarity to reuse model, in the case of ensure the quality of sports training to reduce the number of sports training, so as to improve the efficiency of the sports training.

2. Problem Definition

Without loss of generality, an MOP problem with *n* decision variables and *m* objective functions can be defined as the following

$$\begin{cases} \min F(x) = \left(f_1(x), f_2(x), \ldots, f_m(x)\right)^T \\ s.t.\ \ x \in \Omega \end{cases} \tag{1}$$

In which, $(x_1, x_2, \ldots, x_n)^T \in \Omega$ is the decision vector, Ω is the feasible region space, $f_i : x \to R(i = 1, \ldots, m)$ is the first i-th objective function. For a given MOP, there are often conflicts between the objectives, a solution may be excellent for an objective, and may be poor for the other objectives. Therefore, in general, the optimal solution of MOP is not one solution but a collection of solutions, which is called Pareto optimal solution set.

Definition 1 (Pareto dominance). Assuming that $x, y \in \Omega$ is the feasible solution to the problem of MOP, which can be called x Pareto dominance y (recorded as $x \prec y$), if and only if

$$\forall i = 1, 2, \ldots, m,\ f_i(x) \le f_i(y) \wedge F(x) \ne F(y) \tag{2}$$

Definition 2 (Pareto optimal solution set). The Pareto optimal solution set (Pareto set, referred to as “PS” for short) is the collection of all Pareto optimal solutions:

$$PS = \left\{x \middle| \neg\exists y \in \Omega : y \prec x\right\} \tag{3}$$

Definition 3 (Pareto optimal front) Pareto optimal front (Pareto front, referred to as “PF” for short) is the projection of the Pareto optimal solution set in the objective space:

$$PF = \left\{F(x) \middle| x \in PS\right\} \tag{4}$$

The continuous MOP problem of Pareto solution set not only has the topology structure and the optimal solution for the single objective optimization problem, but also has proven that it has the following features of regularity properties [40]:

Theorem 1. Under certain conditions, for a continuous MOP problem that contains *m* objectives, the Pareto optimal solution set in the solution space and the objective space are both the piecewise continuous dimensional manifold of $(m-1)$.

The theorem shows that continuous MOP problem of Pareto optimal solution is not mixed and disorderly in the distribution, but shows some pattern. Thus, MOEA can through to the Pareto optimal solution collector shape approximation to indirectly realize the

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Pareto optimal solution set of approximation.

3. Sports Training Methods Based on the Mixture Kalman Probability Model

3.1. Sports Training Method Framework Based on the Decomposition

Sports training method based on the strategy of decomposition MOBC [11] is a kind of new MOEA algorithm framework. MOBC algorithm is an effective extension of traditional decomposition methods, and the traditional method, the MOBC will a MOP first broken down into a set of single objective optimization problem (a problem); Is different from the traditional method, the MOBC through cooperation between the sub-problems to optimize this group of sub-problems, at the same time to obtain the Pareto optimal solution set of a looming. Compared with the traditional method for solving the problem of child serial way, the MOBC this parallel algorithm for solving the sub-problem strategy can effectively improve the search efficiency. At the same time, the MOBC framework can naturally have the optimization on all kinds of the single objective, and local search method is used as the search operator. Figure 1 in 2 objective optimization problem, for example shows the MOBC algorithm the basic idea, In which, the sub-problem has 4 neighbor children problem son 2 ~ 6. The following through two important concepts in MOBC algorithm to further introduce the algorithm.

Definition 4 (Sub-problem) - An MOP problem $\min F(x)$ can be transformed into a set of sub-problems $\min g^i(x), i = 1, 2, \ldots, N$, each sub-problem of optimal solutions corresponding to the original MOP's Pareto optimal solution of the problem.

If the sub-problem definition is reasonable, this set of sub-problems of the optimal solution can be used as a Pareto optimal solution set of approximation. Usually, algorithms based on the strategy of decomposition by a given a set of weighted vectors $\left(w_1^i, w_2^i, \ldots, w_m^i\right)^T$, $i = 1, 2, \ldots, N$ is applied to define the sub-problem. In this paper, Chebyshev method is applied to define the sub-problem:

$$g^i(x) = g\left(x \middle| w^i, z^*\right) = \max_{j=1,\ldots,m} \left\{ w_j^i \left(f_j(x) - z_j^* \right) \right\} \tag{5}$$

In which, $z^* = \left(z_1^*, z_2^*, \ldots, z_m^*\right)$ is the ideal point of the MOP problem, namely, $z_j^* < \min_{x \in \Omega} f_j(x), j = 1, 2, \ldots, m$, which can prove that: For the MOP problem, every Pareto optimal solutions corresponding to a Chebyshev sub-problem.

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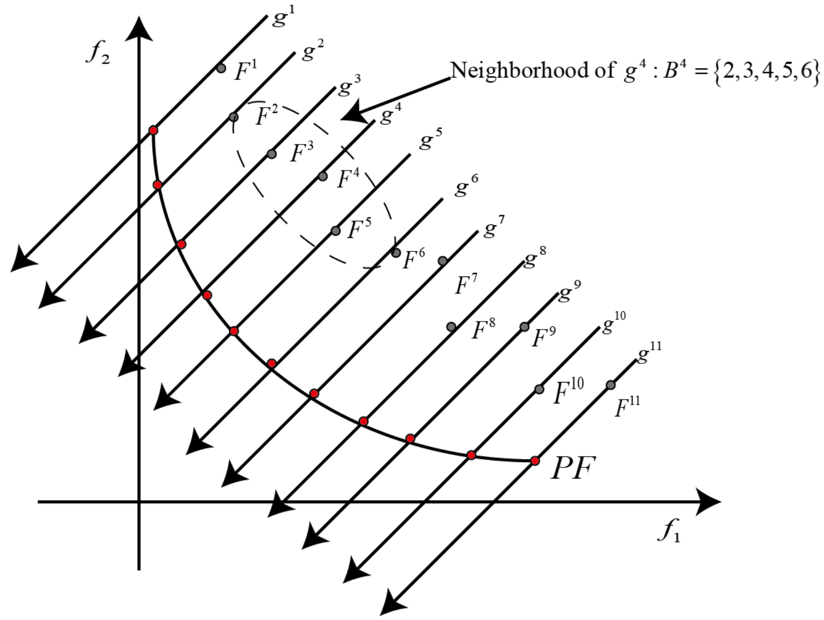

Figure 1 MOBCAlgorithm Diagram

Weight vector $w^i = \left(w_1^i, w_2^i, ..., w_m^i\right)$ shall meet the condition $0 \leq w_j^i \leq 1$ and $j = 1, 2, ..., m \wedge \sum_{j=1}^{m} w_j^i = 1$. In this article, N weight vectors are uniformly selected in the weighting vector space in advance, and for the specific algorithm literature [11] can be referred to. It should be pointed out that, given a set of uniform weight vector may not be able to obtain the optimal solution of a set of uniform distribution. The choice of weight vector is closely related to the shape of the PF. Therefore, how to dynamically adjust the weight vector in the operation process to obtain the optimal solution of uniform distribution, which is currently a hot research topic in the MOBC algorithm design.

Defined 5 (Neighborhood sub-problem (neighborhood).) The neighborhood sub-problem of a sub-problem $\min g^i$, is a set of K sub-problems $\left\{\min g^i \,\middle|\, j = \{i_1, i_2, ..., i_K\}\right\}$ that are of the most similarity. The MOBC by weighting vector distance defines the similarity of sub-problems, the weight vector distance is smaller, and the corresponding sub-problem is similar. Superscript i_j represents the weight vector that w^{i_j} is the j-th close to w^i. Obviously, $i_1 = i$, that is, the sub-problem is the most similar to itself.

The neighborhood sub-problem application in MOBC algorithm is mainly reflected in two aspects: (1) The parent body of the recombination operator mainly comes from the neighborhood. As the parent body apart, can avoid to a certain extent in section 2.2 the restructuring of operator problems. (2) The new individual not only updates its father, but also update its neighborhood individuals, making good new individual retain as much as possible to the next generation. It is as the sub-problems mutual cooperation in the restructuring and update operations MOBC algorithm can simultaneously optimize all sub-problems.

The early version of MOBC adopts the static neighborhood. Predictably, the algorithm run at different stages, different MOP problem requires a different neighborhood. How the execution of the algorithm to dynamically adjust the neighborhood in order to obtain a better algorithm performance, which is also worth studying in the MOBC algorithm design.

3.2. Recombination Operator Based on the Mixture Kalman Probability Model

The estimation of distribution algorithm (Referred to as “EDA” for short) is a kind of new evolutionary algorithm, and the algorithms for EA restructuring such as mixture mutation operator is used to produce

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offspring in a different way, EDA using a probability model to describe the distribution of the current population, and the sampling of the model to produce offspring. Classic recombination operator using local information to produce offspring, individual and EDA algorithm based on the global distribution information, group to produce offspring, therefore, the EDA algorithm can guide the search process from the global point of view.

The Kalman model is one of the most widely researched and applied probability models. A multiple Kalman distribution of random variables $x=(x_1,x_2,...,x_n)^T$ can be represented as the following

$$x \sim N(\mu,\Sigma) \quad (6)$$

In which, μ is the mean vector, Σ is the covariance matrix. And the corresponding probability density function is expressed as a random variable in the following

$$p(x)=(2\pi)^{-\frac{n}{2}}|\Sigma|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right\} \quad (7)$$

For a given set of data $x^1,x^2,...,x^K$, the mean vector and covariance matrix estimation is as follows:

$$\mu=\frac{1}{K}\sum_{k=1}^{K}x^k \quad (8)$$

$$\Sigma=\frac{1}{K-1}\sum_{k=1}^{K}(x^k-\mu)(x^k-\mu)^T \quad (9)$$

Algorithm 1. Multiobjective Kalman model sampling $x=GaussianSample(\mu,\Sigma)$

Step1. Adopt Cholesky decomposition covariance matrix for the lower triangular matrix A, which meets $\Sigma=AA^T$.

Step2. Generate single factor Kalman distribution vector $y=(y_1,y_2,...,y_n)^T$, in which, $y_j \sim N(0,1), j=1,2,...,n$ conforms with unit Calman distribution.

Step3. Let $x=\mu+Ay$.

The analysis in Section 2 has shown that a single Kalman probability model is not applicable for the MOP problem, so we need to consider structure mixture Kalman model. A direct improvement method is: first the clustering analysis method is adopted to group will be divided into multiple categories, and then construct Kalman model respectively for each category. The characteristics of this method is not fully consider the MOP, so for some problems to solve the low efficiency. Mixture Kalman model (sub-problem) for each individual to construct the Kalman model, and the probability density function is expressed as the following

$$p(x)=\sum_{i=1}^{N}\frac{1}{K}p^i(x) \quad (10)$$

In which, $p^i(x)$ stands for the corresponding probability density function problem; μ^i and Σ^i are the corresponding average vector and covariance matrix of $p^i(x)$, the numerical value is calculated from the i-th sub-problems corresponding neighborhood problems. Obviously, for each sub-problem to calculate the mean and covariance matrix, and through large amount of calculation algorithm of Figure 4 sampling new individuals. This section will through the neighborhood sub-problems

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reuse of covariance matrix to reduce the complexity of mixture Kalman model sampling.

To the i-th $i(i=1,2,...,N)$ sub-problem, construct its probability density function as the following:

$$p^i(x)=(2\pi)^{-\frac{n}{2}}\left|\Sigma^i\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(x-\mu^i\right)^T\left(\Sigma^i\right)^{-1}\left(x-\mu^i\right)\right\} \quad (11)$$

In which, the mean vector is expressed by the current best solution of the i-th sub problem, namely, $u^i=x^i$; the covariance matrix Σ^i is calculated by neighborhood sub-problem or directly applies the neighborhood sub-problem corresponding covariance matrix.

4. Experiment Simulation

To test the effectiveness of the proposed MOPE algorithm, this paper adopts 9 test functions that are of complicated PS: F1 ~ F9. In which, F6 has three objective functions, and the other problems have two objective functions; F6 and F9 have non-convex PF, while the other problems have the convex PF; F7 and F8 are the multimodal problems, with multiple local optimal PF.

Due to the fact that the theoretical optimal solution can be obtained for the athletes applied, this paper applies IGD as the evaluation index. $IGD\left(PF,PF^*\right)$ calculates the solution set PF and the distance between the optimal solution set PF *.

$$\begin{cases} IGD\left(PF,PF^*\right)=\dfrac{1}{|PF|}\sum\limits_{x\in PF}d\left(x,PF^*\right) \\ d\left(x,PF^*\right)=\min\limits_{y\in PF^*}\|x-y\| \end{cases} \quad (12)$$

In which, $|\cdot|$ stands for the number of collection points, $\|\cdot\|$ stands for the vector mode. When PF is approximate to PF *, IGD can simultaneously measure the degree of diversity and convergence of PF. In the experiments, the 2 objectives, we select from PF * 500 evenly distributed points on behalf of the PF *, and select 990 delegates for the 3 objective problem.

This paper compares the NSGA II - DE [13], MOBC - DE [13] and MOPE [12] with the method proposed in this paper through the experiment. In order to distinguish, the methods in literature [12] is recorded as MOPE1, and the improved method proposed in this paper is recorded as MOPE2. These methods parameter Settings are as follows: DE operator control parameters $CR=1.0, F=0.5$, the parameters of polynomial mutation operator $\eta=20, P_m=1/n$. Other parameters of MOBC - DE are as follows: the neighborhood size, neighborhood search probability, sub-problems update number. MOPE1 other parameters as follows: on 2 objective problem, neighborhood size; Problem on three objectives, neighborhood size, neighborhood search probability, sub-problems update number. MOPE2 other parameters as follows: the neighborhood size $T=20$, neighborhood search probability $\delta=0.9$, and sub-problems update number $n_r=2$.

For all the algorithms, the 2 objective problem population size $N=299$, 3 objective problem population size $N=595$, and the algorithm executes 500 generations before suspension. The problem independent variable dimension $n=30$ (F1 ~ F5, F9) or (F6 to F8). For each athlete, MOPE is executed for 30 times, MOBC-DE and NSGA-II-DE are executed for 20 times. MOPE1 and MOPE2 are programmed by the application of Matlab, while MOBC - DE and NSGA - II - DE are programmed by the application of C + +.

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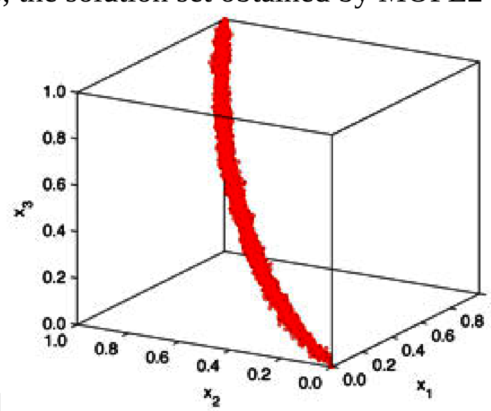
4.1. MOPE2 Experimental Results and Analysis

In Table 1, it shows the statistical results of the IGD index of MOPE2 on 9 test function computation results. As can be seen from the table: with the increase of computing algebra, IGD metric values significantly decrease, especially with significant change between generation 100 and generation 200. After 500 generations of computing, the smaller IGD mean value can all be obtained. These statistical results show that for most athletes, MOPE2 can find stability in a smaller amount of calculation and the quality of the solution set. MOPE2 could fall into local optimum and cause IGD mean and mean square error is bigger. MOPE2 can obtain good results, which indicates that MOPE2 has the multimodal problem solving capability.

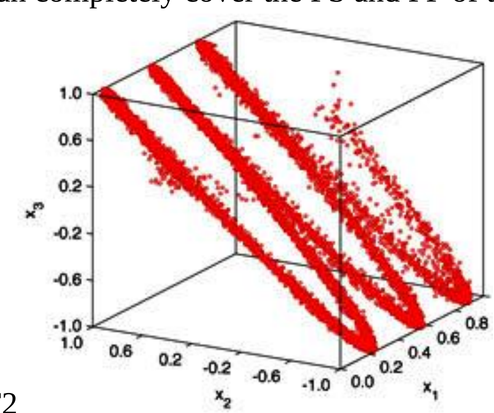
Table 1. Generation 100, 200, 300, 400 and 500 MOPE2 results IGD measurement mean and mean square error

	100	200	300	400	500
F1	0.0024 （0.0001）	0.0017 （0.0000）	0.0015 （0.0000）	0.0014 （0.0000）	0.0014 （0.0000）
F2	0.0551 （0.0166）	0.0256 （0.0117）	0.0106 （0.0060）	0.0048 （0.0017）	0.0033 （0.0003）
F3	0.0252 （0.0045）	0.0083 （0.0022）	0.0043 （0.0011）	0.0031 （0.0006）	0.0025 （0.0002）
F4	0.0377 （0.0072）	0.0161 （0.0033）	0.0096 （0.0014）	0.0059 （0.0016）	0.0040 （0.0013）

Figure 2 shows the final solution set obtained by MOPE2. The optimal PF obtained for F1 ～ F4 can approximate the real PF*; As can be seen from Figure 2, the complicated PS of these problem has brought difficulties to algorithm, but most MOPE2 can effectively approximate PS. Figure 2 shows that: in the case of run multiple times, the solution set obtained by MOPE2 can completely cover the PS and PF of these problems.



F1



F2

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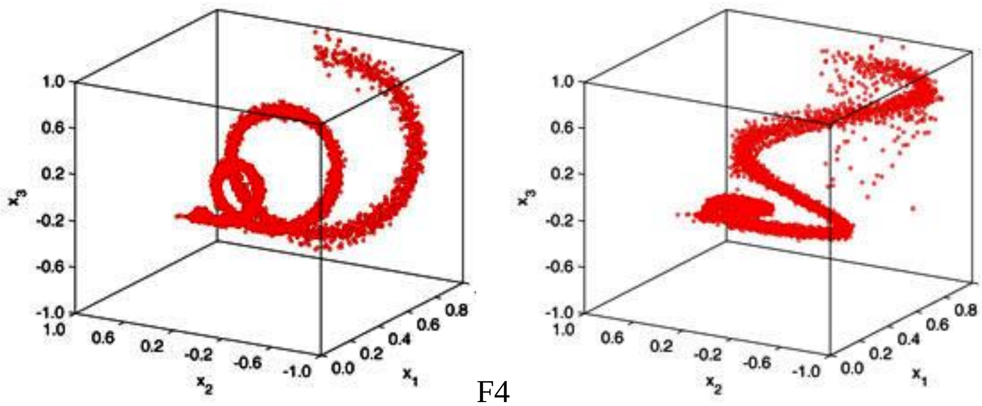

F3 F4

Figure 2. Final Solution Set of MOPE2 Algorithm

4.2. Algorithm Contrast Experiment

This section compares the MOBC algorithm based on mixture Kalman model and the NSGA-II-DE and MOBC-DE which are proposed in literature [13]. Table 2 is the final results of statistical IGD mean and mean square error of the 4 kinds of algorithms.

Table 2. Statistical IGD mean and mean square error of the results of the 4 kinds of comparison algorithms

	MOBC-DE[13]	NSGA-II-DE[13]	MOPE1[12]	MOPE2
F1	0.0015 （0.0000）	0.0044 （0.0000）	**0.0009** （0.0000）	0.0014 （0.0000）
F2	**0.0028** （0.0004）	0.0349 （0.0066）	0.0040 （0.0016）	0.0033 （0.0003）
F3	0.0068 （0.0099）	0.0296 （0.0030）	**0.0024** （0.0005）	0.0025 （0.0002）
F4	**0.0040** （0.0014）	0.0288 （0.0021）	0.0046 （0.0024）	**0.0040** （0.0013）

The statistical results show that:

- MOBC - DE in F2, F4 can achieve the best results;
- MOPE1 in F1, F3 can the best results;
- MOPE2 in F4 can best results;
- For the comparison of MOBC - DE, MOPE2 and MOPE1, NSGA II - DE cannot achieve the best results.

It should be pointed out that:

- For F2 and MOPE2, the result of the IGD mean is 1.18 times of the mean obtained by MOBC - DE;
- For F3 and MOBC-DE, the result of the IGD mean is 2.72 times of the mean obtained by MOPE2;
- For F1 and F4, the two algorithms have the similar results.

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5. Conclusion

The multiobjective algorithm that is based on the strategy of decomposition is a new type algorithm framework to solve the sports training method. At present, most MOBC algorithms adopt the traditional recombination operator. In view of this situation, this paper has certificated or analyzed the defects of several typical recombination operators under the current sports training methods. On this basis, the MOBC algorithm based on the mixture Kalman model MOPE is put forward. MOPE applies the mixture Kalman model to obtain the distribution of the population, and perform the sampling on the distribution to obtain the new individuals; at the same time, a greedy strategy is adopted to update the group. The experimental analysis results show that, the mixture Kalman model can effectively extract the structure information of the population, which can achieve approximation more effectively to the complicated PS compared to the DE operator; and the greedy update strategy can promote the evolution of the population more effectively.

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