TEST Engineering & Management

July-August 2020
ISSN: 0193-4120 Page No. 4162 - 4167

Variety of Rational Resolving Sets of Power of a Cycle

M. M. Padma and M. Jayalakshmi
Department of Mathematics, Dr. Ambedkar Institute of Technology, Bengaluru, Karnataka, India

Article Info Volume 83
Page Number: 4162 - 4167
Publication Issue:
July - August 2020

Article History
Article Received: *25 April 2020*
Revised: *29 May 2020*
Accepted: *20 June 2020*
Publication: *10 August 2020*

Abstract:
To discover the actual route and to determine the position of a vertex in the network, we need to select the landmarks by making certain local measurement at the smallest subsets of the nodes. Since each of these measurements are potentially quite costly, the objective here is to minimize the number of measurements which still discover the whole graph. A subset S of vertices of a graph G is called a rational resolving set of G if for each pair $u, v \in V - S$, there is a vertex $s \in S$ such that $d(u/s) \neq d(v/s)$, where $d(x/s)$ denotes the mean of the distances from the vertex s to all those $y \in N[x]$. A rational resolving set denoted by r_r set, having minimum cardinality is a rational metric basis and its cardinality is the lower r_r number, denoted by $l_{r_r}(G)$. The maximum cardinality of a minimal r_r set is called the upper r_r number of G, denoted by $u_{r_r}(G)$. In this paper varieties of minimal rational resolving sets of a graph G are defined on the basis of its compliments, called the lower and upper r_r, r_r^*, R_r, R_r^* numbers and discussed their optimality in power of a cycle.

Keywords: *Power of a Graph, Resolving Set, Rational Resolving Set, Metric Dimension, Rational Metric Dimension.*

AMS Subject Classification number: $05C12, 05C38$

§ 1. INTRODUCTION

Due to the global exponential growth, it is hard to obtain the accurate map of the internet and such networks are represented by a graph with nodes and links, is a prerequisite when investigating the properties of internet. To discover the actual route and to determine the position of a vertex in the network, we need to select the landmarks by making certain local measurement at the smallest subsets of the nodes. Since each of these measurements are potentially quite costly, the objective here is to minimize the number of measurements which still discover the whole graph. We formulate this problem by defining the rational metric dimension in such a way that the distance of the vertex from the landmark and the distances of its neighborhood vertices from the landmark are considered.

For each vertex u of a the graph G, $N(u) = \{x : ux \in E(G)\}$ denotes the open neighborhood of u and $N[u] = N(u) \cup \{u\}$ denotes the closed neighborhood of u. Let $d(u, v)$ be the length of a shortest path between u and v. We use the standard terminology, the terms not defined here may be found in [1, 3].

All the graphs considered here are undirected, finite, connected and simple. A subset S of the vertex set V of a connected graph G is said to be a resolving set of G if for every pair of vertices $u, v \in V - S$ there exists a vertex $w \in S$ such that $d(u, w) \neq d(v, w)$. The minimum cardinality of a resolving set S of G is called the metric dimension of a graph G and is denoted by $\beta(G)$. The metric dimension was defined by F. Harary and R. A. Melter [2], and indepently by P. J. Slater [6]. B. Sooryanarayana et al. [8, 9] obtained many results on metric dimension.

In 2014, A. Raghavendra et al. [7] introduced rational metric dimension of graphs. M. M. Padma and M. Jayalakshmi [4, 5], introduced the concept of r_r, r_r^*, R_r, R_r^* sets of graphs. Wong and Copper-smith [10] defined a circulant graph as a generalization to the double loop network and is used for the design of computer and communication network due to its optimal fault tolerance and routing capabilities. In this paper, various classes of rational resolving sets and rational metric dimension for the circulant graph $C_n(1, 2, \ldots, k)$ for certain k are discussed.

§ 2. On classes of rational resolving sets of a graph

Consider a graph $G(V, E)$. For $u \in V$, associate a vector with respect to a subset $S = \{s_1, s_2, s_3, \ldots, s_k\}$ of V, by $\Gamma(u/S) = (d(u/s_1), d(u/s_2), \ldots, d(u/s_k))$, where $d(u/v) = \frac{\Sigma_{u_i \in N[u]} d(u_i, v)}{deg(u)+1}$. Then the subset S is said to be a rational resolving set which is also called an r_r set if $\Gamma(x/S) \neq \Gamma(y/S)$ for all $x, y \in V - S$. The minimum cardinality of a rational resolving set S is called the rational metric dimension and is denoted by $\mathrm{rmd}(G)$ or $l_{r_r}(G)$. An r_r set of G is said to be minimal if no subset of it is an r_r set. Clearly minimum cardinality of a minimal r_r set is $l_{r_r}(G)$, also called the lower r_r number and the maximum cardinality of a minimal r_r set of graph G is called the upper r_r number of G, denoted by $u_{r_r}(G)$. A subset S of $V(G)$ is said to be an r_r^* set if S is an r_r set and $\overline{S} = V - S$ is also an r_r set. The minimum and the maximum cardinality of a minimal r_r^* set of graph G are called, respectively, the lower r_r^* number and upper r_r^* number of G and are denoted by $l_{r_r^*}(G)$ and $u_{r_r^*}(G)$. A subset S of $V(G)$ is said to be an R_r set if S an r_r set and $\overline{S}$ is not an r_r set. The minimum and maximum cardinality of a minimal R_r sets of G are called, respectively, the lower and upper R_r number of G and are denoted by $l_{R_r}(G)$ and $u_{R_r}(G)$. A subset S of $V(G)$ is said to be an R_r^* set if both S and $\overline{S}$ are not r_r sets. The minimum and maximum cardinality of a minimal R_r^* sets of G are called, respectively, lower and upper R_r^* number of G and are denoted by $l_{R_r^*}(G)$ and $u_{R_r^*}(G)$.

§ 3. On classes of rational resolving sets of power of a cycle

Let $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_n on n vertices in order. The k^{th} power of the graph G, denoted by G^k, defined on vertex set of G and two distinct vertices u and v of G are adjacent in G^k if and only if their distance in G is at most k. Circulant graph $C_n(1,2,3,\dots,k)$ is k^{th} power of a cycle C_n with the vertex set $V(C_n^k) = V(C_n) = \{v_1, v_2, \dots, v_n\}$.

Remark 3.1. The graph C_n^k is defined for any $k \in Z^+$ and $C_n^k = C_n^{k+1} = K_n$ whenever $k \geq \left\lfloor\frac{n}{2}\right\rfloor \left(\text{dia}(C_n) = \left\lfloor\frac{n}{2}\right\rfloor\right)$. If k = 1, then $C_n^k = C_n$ and if $k = \left\lfloor\frac{n}{2}\right\rfloor$, that is $k = \frac{n}{2}$ when n is even; $k = \frac{n-1}{2}$, when n is odd, then $C_n^k = K_n$ and in both the cases r_r, r_r^*, R_r, R_r^* are discussed in article [4] · For any graph G, we use the convention that if any of r_r, r_r^*, R_r, R_r^* sets does not exist, then their cardinality is zero.

Lemma 3.2. *For the graph* C_{2k+2}^k , $rmd(C_{2k+2}^k) = \frac{n}{2}$.

Proof. Let $n = 2k+2$ and v_1 be any vertex of C_n^k. Then $d(v_1, v_i) = 1$ for every i with $2 \leq i \leq n$ except $i = \frac{n+2}{2}$ and $d(v_1, v_{\frac{n+2}{2}}) = 2$ which implies,

$$d(v_i/v_1) = \begin{cases} \frac{2k}{2k+1}, & \text{if } \ i = 1. \\ \frac{2k+2}{2k+1}, & \text{if } \ i = \frac{n+2}{2}. \\ 1, & \text{otherwise} \end{cases}$$

Hence $d(v_i/v_1)$ remains same for $n-2$ vertices except for $i = 1, \frac{n}{2}+1$. Let $v_j \neq v_1$ be any other vertex. Then $d(v_i/v_j)$ is the same for $n-2$ vertices except for $i = j$, $j + \frac{n}{2}$ which implies $\Gamma(v_i/\{v_1, v_j\})$ remains same for $n-4$ vertices. Continuing in a similar way minimum $\frac{n}{2}$ number of vertices are required to rational resolve C_n^k. Thus any subset of $V(C_n^k)$ containing $\frac{n}{2}$ vertices can rational resolve C_n^k and is minimal. Hence $rmd(C_n^k) = \frac{n}{2}$.

Theorem 3.3. *For the graph* C_n^k *with* $k = \frac{n-2}{2}$,

i. $l_{r_r}(C_n^k) = u_{r_r}(C_n^k) = \frac{n}{2}$.

ii. $l_{r_r^*}(C_n^k) = u_{r_r^*}(C_n^k) = \frac{n}{2}$.

iii. $l_{R_r}(C_n^k) = u_{R_r}(C_n^k) = \frac{n}{2} + 1$.

iv. $l_{R_r^*}(C_n^k) = u_{R_r^*}(C_n^k) = 0$.

Proof. Let $k = \frac{n-2}{2}$.

i. From Lemma 3.2, any subset of $V(C_n^k)$ containing $\frac{n}{2}$ vertices can rational resolve C_n^k and is minimal with minimum and maximum cardinality. Hence $l_{r_r}(C_n^k) = u_{r_r}(C_n^k) = \frac{n}{2}$.

ii. Any subset of $V(C_n^k)$ containing $\frac{n}{2}$ number of vertices are required to rational resolve C_n^k. So for any subset S of $V(C_n^k)$ containing $\frac{n}{2}$ elements, $\overline{S}$ will also contain $\frac{n}{2}$ vertices. Therefore both S and $\overline{S}$ are r_r sets with minimum and maximum cardinality and hence $l_{r_r^*}(C_n^k) = u_{r_r^*}(C_n^k) = \frac{n}{2}$.

iii. From Lemma 3.2, any r_r set has to contain minimum $\frac{n}{2}$ elements, if S contain $\frac{n}{2}+1$ elements, then $\overline{S}$ will contain less than $\frac{n}{2}$ elements which imply S is an r_r set and $\overline{S}$ is not an r_r set. Therefore S is a minimal R_r set with minimum and maximum cardinality and hence $l_{R_r}(C_n^k) = u_{R_r}(C_n^k) = \frac{n}{2} + 1$.

iv. From Lemma 3.2, a subset of $V(C_n^k)$ containing minimum $\frac{n}{2}$ elements is an r_r set. Hence for any subset S of $V(C_n^k)$, either S or $\overline{S}$ contain minimum $\frac{n}{2}$ elements, so that, S or $\overline{S}$ is an r_r set. Therefore, there exists no R_r^* set for C_n^k, which imply $l_{R_r^*}(C_n^k) = u_{R_r^*}(C_n^k) = 0$.

Lemma 3.4. *For any integer* $n \geq 4$, $rmd(C_n^2) = \begin{cases} n-1 & if\ n = 4, 5. \\ 3 & if\ n = 6. \\ 2 & if\ n > 6. \end{cases}$

Proof. Follows with various values of n. If $n = 4, 5$, then

July-August 2020
ISSN: 0193-4120 Page No. 4162 - 4167

$C_n^2 = K_n$, a complete graph and its rational metric dimension is discussed in [4]. If $n = 6$, then $n = 2k + 2$ and hence by the Lemma 3.2, $rmd(C_n^2) = \frac{n}{2} = 3$. For $n > 6$, the following cases arises.

Case i: $n \equiv 1, 2, 3 (\text{mod } 4)$.

In this case, $d(v_1, v_i) \leq d(v_1, v_{i-1})$ and $d(v_1, v_{i-2}) < d(v_1, v_i)$ for every i with $3 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 1$, which imply $d(v_i/v_1)$ is strictly increasing for every i with $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$ and by symmetry $d(v_i/v_1) = d(v_{n-(i-2)}/v_1)$ for every i with $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$. Hence minimum two vertices are required to rational resolve C_n^2 and for any v_j, $j \neq 1$, $\Gamma(v_i/\{v_1, v_j\})$ is different for distinct v_i's. Therefore, $rmd(C_n^2) = 2$.

Case ii: $n \equiv 0 (\text{mod } 4)$.

In this case, $d(v_1, v_i) \leq d(v_1, v_{i-1})$ and $d(v_1, v_{i-2}) < d(v_1, v_i)$ for every i with $3 \leq i \leq \frac{n}{2} + 1$, which imply $d(v_i/ v_1)$ is strictly increasing for every i with $1 \leq i \leq \frac{n}{2}$. By symmetry $d(v_i/ v_1) = d(v_{n-(i-2)}/v_1)$ for every i with $2 \leq i \leq \frac{n}{2}$, which imply minimum two vertices are required to rational resolve C_n^2 and $d(v_{\frac{n}{2}}/v_1)= d(v_{\frac{n}{2}+1}/v_1)= d(v_{\frac{n}{2}+2}/v_1)$, imply two adjacent vertices or diagonally opposite vertices of C_n cannot resolve C_n^2. Also for any v_j, which is not adjacent to v_1 in C_n or not diagonally opposite to v_1, $\Gamma(v_i/\{v_1, v_j\})$ remains different for all v_i's. Therefore $rmd(C_n^2) = 2$.

Theorem 3.5. *For the graph C_n^2, with $n > 6$ and $n \equiv 1, 2, 3$ (mod 4),*

i. $l_{rr}(C_n^2) = u_{r_r}(C_n^2) = 2$.

ii. $l_{r_r^*}(C_n^2) = u_{r_r^*}(C_n^2) = 2$.

iii. $l_{R_r}(C_n^2) = u_{R_r}(C_n^2) = n - 1$.

iv. $l_{R_r^*}(C_n^2) = u_{R_r^*}(C_n^2) = 0$.

Proof: When $n \equiv 1, 2, 3$ (mod 4),

i. Any subset of $V(C_n^2)$ containing two vertices can rational resolve C_n^2 and is minimal with minimum and maximum cardinality. Hence $l_{r_r}(C_n^2) = u_{r_r}(C_n^2) = 2$.

ii. Any subset of $V(C_n^2)$ containing two vertices are required to rational resolve C_n^2, for any subset S of $V(C_n^2)$ with $|S| = 2$, $\overline{S}$ contain minimum two vertices. Therefore both S and $\overline{S}$ are r_r sets and hence S is minimal r_r^* set with minimum and maximum cardinality, which imply $l_{r_r^*}(C_n^2) = u_{r_r^*}(C_n^2) = 2$.

iii. Any r_r set has to contain minimum two elements, if S contain $n - 1$ elements then $\overline{S}$ will contain only one element which imply S is an r_r set and $\overline{S}$ is not an r_r set. Therefore S is a minimal R_r set with minimum and maximum cardinality and hence $l_{R_r}(C_n^2) = u_{R_r}(C_n^2) = n - 2$.

iv. Any r_r set has to contain minimum two elements, for any subset S of $V(C_n^2)$, either S or $\overline{S}$ contain minimum two vertices. Therefore there exists no R_r^* set for C_n^2 and hence $l_{R_r^*}(C_n^2) = u_{R_r^*}(C_n^2) = 0$.

Theorem 3.5. *For the graph C_n^2, with $n > 6$ and $n \equiv 0 (\text{mod } 4)$.*

i. $l_{r_r}(C_n^2) = u_{rr}(C_n^2) = 2$.

ii. $l_{r_r^*}(C_n^2) = u_{r_r^*}(C_n^2) = 2$.

iii. $l_{R_r}(C_n^2) = u_{R_\Gamma}(C_n^2) = n - 2$.

iv. $l_{Rr^*}(C_n^2) = u_{R_r^*}(C_n^2) = 0$.

Proof: When $n \equiv 0 (\text{mod } 4)$.

i. A subset of $V(C_n^2)$ containing two non adjacent vertices of C_n or non-diagonal vertices can rational resolve C_n^2 and is minimal with minimum and maximum cardinality. Hence $l_{r_r}(C_n^2) = u_{r_r}(C_n^2) = 2$.

ii. A subset of $V(C_n^2)$ containing two non adjacent vertices of C_n or non diagonal vertices are required to rational resolve C_n^2, if S is such a subset of $V(C_n^2)$ then $\overline{S}$ contain minimum two non adjacent vertices of C_n. Therefore both S and $\overline{S}$ are r_r sets and hence S is a minimal r_r^* set with minimum and maximum cardinality which imply $l_{r_r^*}(C_n^2) = u_{r_r^*}(C_n^2) = 2$.

iii. Any r_r set has to contain minimum two non adjacent vertices of C_n or non diagonal vertices, if $S = \{v_1, v_2, \ldots, v_{n-2}\}$, which contain $n - 2$ elements then $\overline{S} = \{v_{n-1}, v_n\}$. Therefore S is an r_r set and $\overline{S}$ is not an r_r set and hence S is a minimal R_r set with minimum and maximum cardinality which imply $l_{R_r}(C_n^2) = u_{R_r}(C_n^2) = n - 2$.

iv. Any r_r set has to contain minimum two non adjacent vertices of C_n or non diagonal elements, for any subset S of $V(C_n^2)$, either S or $\overline{S}$ can not contain such two vertices. Therefore there exists no R_r^* set for C_n^2 and hence $l_{R_r^*}(C_n^2) = u_{R_r^*}(C_n^2) = 0$.

Lemma 3.6. *For any integer $n \geq 6$,*

$$\text{rmd}(C_n^3) = \begin{cases} n-1 & if\ n = 6, 7. \\ 4 & if\ n = 8. \\ 2 & if\ n > 8. \end{cases}$$

Proof: If $n = 6, 7$ then $C_n^3 = K_n$ and its rational metric dimension is discussed in [4]. If $n = 8$ then $n = 2k + 2$, hence by the Lemma 3.2, $\text{rmd}(C_n^3) = \frac{n}{2} = 4$. If $n = 9$ then rmd $(C_n^3) = 2$ from Figure 1. If $n > 9$ the following cases arises.

Case i: $n \equiv 1 (\text{mod } 6)$ or $n \equiv 2 (\text{mod } 6)$ or $n \equiv 3 (\text{mod } 6)$.

Here $d(v_i/v_1)$ is strictly increasing and by symmetry $d(v_i / v_1) = d(v_{n-(i-2)}/ v_1)$ for every i with $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$. Hence minimum two vertices are required to rational resolve C_n^3 and for any v_j, $j \neq 1$, $\Gamma(v_i /\{v_1, v_j\})$ is different for all v_i's. Therefore $rmd(C_n^3) = 2$.

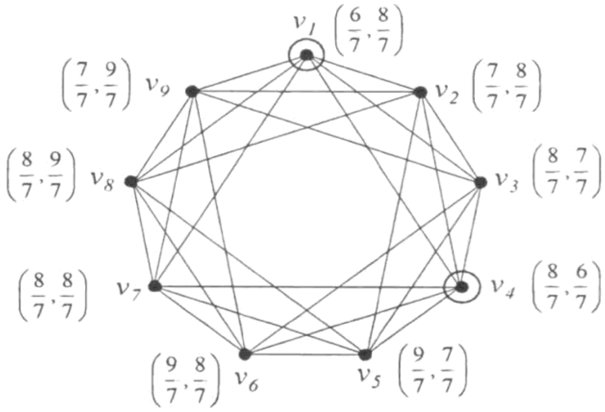

Figure 1: The graph C_9^3 and its 2-element rational metric basis.

Case ii: $n \equiv 0, 4 (\text{mod } 6)$.

Here $d(v_i/v_1)$ is strictly increasing and by symmetry $d(v_i/v_1) = d(v_{n-(i-2)} / v_1)$ for every i with $2 \leq i \leq \frac{n}{2}$ which imply minimum two vertices are required to rational resolve C_n^3. Also $d(v_{\frac{n}{2}} / v_1) = d(v_{\frac{n}{2}+1} / v_1) = d(v_{\frac{n}{2}+2} / v_1)$, which imply two adjacent vertices of C_n or two diagonally opposite vertices can not resolve C_n^3. Hence for any v_j, which is not adjacent in C_n or diagonally opposite to v_1, $\Gamma(v_i/\{v_1, v_j\})$ is different for all v_i's. Therefore $rmd(C_n^3) = 2$.

Case iii: $n \equiv 5 (mod 6)$.

Here $d(v_i/v_1)$ is strictly increasing and by symmetry $d(v_i/v_1) = d(v_{n-(i-2)}/v_1)$ for every i with $2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$ which imply minimum two vertices are required to rational resolve C_n^3. Also, $(v_{\lfloor \frac{n}{2} \rfloor}/v_1) = d(v_{\lfloor \frac{n}{2} \rfloor+1} / v_1) = d(v_{\lfloor \frac{n}{2} \rfloor+2} / v_1) = d(v_{\lfloor \frac{n}{2} \rfloor+3} / v_1)$, which imply vertices v_i, v_j of C_n with $|j - i| \leq 2$ can not resolve C_n^3. Hence for any v_j, with $2 < |j - 1| < \left\lfloor \frac{n}{2} \right\rfloor$, $\Gamma(v_j/\{v_1, v_j\})$ is different for all v_i's. Therefore $rmd(C_n^3) = 2$.

Theorem 3.7. *For the graph C_n^3, with $n > 8$,*

When $n = 9, n \equiv 5$ (mod 6)

i. $l_{r_r}(C_n^3) = 2, u_{r_r}(C_n^3) = 3$.

ii. $l_{r_r^*}(C_n^3) = 2, u_{r_r^*}(C_n^3) = 3$.

iii. $l_{R_r}(C_n^3) = u_{R_r}(C_n^3) = n - 2$.

iv. $l_{R_r^*}(C_n^3) = u_{R_r^*}(C_n^3) = 0$.

when $n \equiv 1, 2, 3 (\text{mod } 6)$,

i. $l_{r_r}(C_n^3) = u_{r_r}(C_n^3) = 2$.

ii. $l_{r_r^*}(C_n^3) = u_{r_r^*}(C_n^3) = 2$.

iii. $l_{R_r}(C_n^3) = u_{Rr} (C_n^3) = n - 1$.

iv. $l_{R_r^*}(C_n^3) = \text{U}_{R_r^*}(C_n^3) = 0$.

and when $n \equiv 0, 4 (\text{mod } 6)$,

i. $l_{r_r}(C_n^3) = 2, u_{r_r}(C_n^3) = 3$.

ii. $l_{r_r^*}(C_n^3) = 2, u_{r_r^*}(C_n^3) = 3$.

iii. $l_{Rr}(C_n^3) = u_{Rr} (C_n^3) = n - 2$.

iv. $l_{R_r^*}(C_n^3) = u_{R_r^*}(C_n^3) = 0$.

Proof. Consider the following cases:

When $n = 9, n \equiv 5$ (mod 6).

i. Any subset $\{ v_i, v_j\}$ of $V(C_n)$ with $2 < |j - i| < \left\lfloor \frac{n}{2} \right\rfloor$ can resolve C_n^3 and is minimal with minimum cardinality which imply $l_{r_r}(C_n^3)= 2$. Also $\{ v_1, v_2 , v_3 \}$ is a minimal r_r set with maximum cardinality, which imply $u_{r_r}(C_n^3) = 3$.

ii. Any subset $\{v_i, v_j\}$ of $V(C_n)$ with $2 < |j - i| < \left\lfloor \frac{n}{2} \right\rfloor$ can resolve C_n^3, if S is such a subset then $\overline{S}$ contain two vertices which are at distance greater than 2, so that both S and $\overline{S}$ are r_r set. Hence S is a minimal r_r^* set with minimum cardinality and therefore $l_{r_r^*}(C_n^3) = 2$. Also if $S = \{v_1, v_2, v_3\}$ then $\overline{S}$ is an r_r set and hence S is a minimal r_r^* set with maximum cardinality and hence $u_{r_r^*}(C_n^3) = 3$.

iii. If $S = \{v_1, v_2, \dots, v_{n-2}\}$ then $\overline{S} = \{v_{n-1}, v_n\}$, which imply S is an r_r set and $\overline{S}$ is not an r_r set. Therefore S is a minimal R_r set with minimum and maximum cardinality and hence $l_{R_r}(C_n^3) = u_{R_r}(C_n^3) = n - 2$.

TEST Engineering & Management

July-August 2020
ISSN: 0193-4120 Page No. 4162 - 4167

iv. Any r_r set has to contain minimum two vertices as mentioned in i**,** for a subset S of $V(C_n^3)$, either S or $\overline{S}$ can not contain such two vertices. Therefore there exists no R_r^* set for C_{n}^3 and hence $l_{R_r^*}(C_n^3) = u_{R_r^*}(C_n^3) = 0$.

When $n \equiv 1, 2, 3 (\mathrm{mod}\ 6)$.

i. Any subset of $V(C_n^3)$ containing two vertices can rational resolve C_n^3 and is minimal with minimum and maximum cardinality. Hence $l_{r_r}(C_n^3) = u_{r_r}(C_{\mathrm{n}}^3) = 2$.

ii. A subset of $V(C_n^3)$ containing two vertices is required to rational resolve C_n^3, for any subset S of $V(C_n^3)$ with $|S| = 2$, $\overline{S}$ contain minimum two vertices. Therefore S is a minimal r_r^* set with minimum and maximum cardinality and hence $l_{r_r^*}(C_n^3) = u_{r_r^*}(C_n^3) = 2$.

iii. If a subset S of $V(C_n^3)$ contain $n-1$ elements then $\overline{S}$ will contain only one element, which imply S is an r_r set and $\overline{S}$ is not an r_r set. Therefore S is a minimal R_r set with minimum and maximum cardinality and hence $l_{R_r}(C_n^3) = u_{R_r}(C_n^3) = n-1$.

iv. For any subset S of $V(C_n^3)$, both S and $\overline{S}$ can not contain less than two elements at the same time. Therefore there exists no R_r^* set for C_n^3 and hence $l_{R_{\mathrm{r}}^*}(C_n^3) = u_{R_{\mathrm{r}}^*}(C_n^3) = 0$.

When $n \equiv 0, 4 (\mathrm{mod}\ 6)$.

i. A subset containing two non adjacent, non diagonal vertices of C_n can rational resolve C_n^3 and is minimal with minimum cardinality. Hence $l_{r_r}(C_n^3) = 2$. Also $\{v_1, v_2, v_{\frac{n}{2}+1}\}$ is minimal r_r set with maximum cardinality which imply $u_{r_r}(C_n^3) = 3$.

ii. A subset containing two non adjacent, non diagonal vertices of C_n are required to rational resolve C_{n}^3, if S is such a subset then $\overline{S}$ contain minimum two non adjacent vertices. Therefore both S and $\overline{S}$ are r_r sets and hence S is an r_{r}^* set with minimum cardinality which imply $l_{r_r^*}(C_n^3) = 2$. Also, if $S = \{v_1, v_2, v_{\frac{n}{2}+1}\}$ then $\overline{S}$ contain minimum two non adjacent vertíces. Therefore both S and $\overline{S}$ are r_r sets and hence S is an r_r^* set with maximum cardinality which imply $u_{r_r^*}(C_{\mathrm{n}}^3) = 3$.

iii. Any r_r set has to contain minimum two non adjacent, non diagonal vertices of C_n, if $S = \{v_1, v_2, \ldots, v_{n-2}\}$, which contain $n-2$ elements then $\overline{S} = \{v_{n-1}, v_n\}$, which imply S is an r_r set and $\overline{S}$ is not an r_r set. Therefore S is a minimal R_{r} set with minimum and maximum cardinality and hence $l_{R_r}(C_n^3) = u_{R_r}(C_n^3) = n-2$.

iv. For any subset $of\ V(C_n^3)$, either S or $\overline{S}$ contain non adjacent vertices. Therefore there exists no R_r^* set for C_n^3 and hence $l_{R_r^*}(C_n^3) = u_{R_r^*}(C_n^3) = 0$.

Theorem 3.8. *For the power graph C_n^k with $n \geq 3k$, $rmd(C_n^k) = 2$.*

Proof. Let v_1 be any vertex of C_n^k. Then $rmd(C_n^k) > 1$ as $d(v_{\mathrm{i}} / v_1) = d(v_{\mathrm{n-(i-2)}} / v_1)$ for every i with $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$. Also $d(v_1, v_{\mathrm{i}}) = d(v_1, v_{n-(i-2)}) = 1$ for all i with $2 \leq i \leq k+1$, $d(v_1, v_i) = d(v_1, v_{n-(i-2)}) = 2$ for all i with $k+2 \leq i \leq 2k+1$. So, in general $d(v_1, v_i) = d(v_1, v_{n-(i-2)}) = \left\lfloor \frac{n}{2k} \right\rfloor$ for all i with $(\left\lfloor \frac{n}{2k} \right\rfloor - 1)k + 2 \leq i \leq \left\lfloor \frac{n}{2k} \right\rfloor k + 1$ and $d(v_1, v_{\mathrm{i}}) = \left\lfloor \frac{n}{2k} \right\rfloor + 1$ for all i with $\left\lfloor \frac{n}{2k} \right\rfloor k + 2 \leq i \leq n - \left\lfloor \frac{n}{2k} \right\rfloor k$, so that $n - 2\left\lfloor \frac{n}{2k} \right\rfloor k - 1$ vertices are at distance $\left\lfloor \frac{n}{2k} \right\rfloor + 1$ from the vertex v_1, which results, for $\left(\left\lfloor \frac{n}{2k} \right\rfloor - 1\right)k + n - 2\left\lfloor \frac{n}{2k} \right\rfloor k - 1 = n - \left(\left\lfloor \frac{n}{2k} \right\rfloor + 1\right)k - 1 = l(say)$ vertices v_i from v_2, $d(v_i/v_1)$ is strictly increasing. That is $d(v_{\mathrm{i}}/v_1)$ is strictly increasing along with i for every i with $2 \leq i \leq l+1$ and may be equal to that of the adjacent vertices for the remaining vertices. Choose v_{k+1} as the second vertex to rational resolve. Then by the similar argument $d(v_{k+i}/v_{k+1})$ is strictly increasing along with i for every i with $2 \leq i \leq 2 + l - 1$ where $k + 2 + l - 1 = k + 1 + n - (\left\lfloor \frac{n}{2k} \right\rfloor + 1)k - 1 = n - \left\lfloor \frac{n}{2k} \right\rfloor k$ and may be equal to that of the adjacent vertices for the remaining vertices. Hence we have $(v_i/v_1) = d(v_{n-(i-2)}/v_1)$, but $d(v_i/v_{k+1}) \neq d(v_{n-(i-2)}/v_{k+1})$ for every i with $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$ which imply $\Gamma(v_i/\{v_1, v_{k+1}\})$ is different for all v_i's. Therefore $\mathrm{rm}d(C_n^k) = 2$.

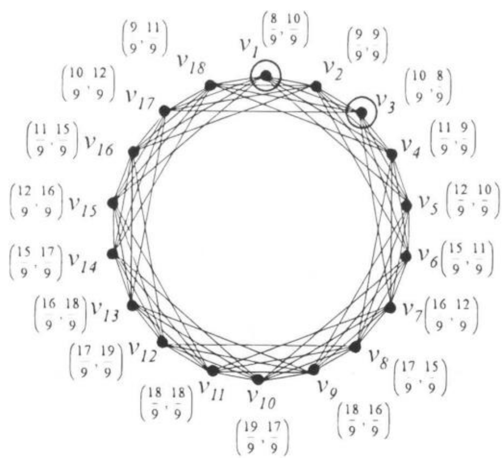

Figure 2: The 2-element rational metric basis of C_{18}^4.

§ 4. Conclusion and Scope

In the study of rational metric dimension various classes of rational metric dimension of square and cube of a cycle is obtained. Also the rational metric dimension of power graph of a cycle C_n^k when $n = 2k + 2$ and $n \geq 3k$ is obtained.

The following are some interesting problems for further investigation.

Problem 1: For the power graph C_n^k with $2k + 3 \leq n \leq 3k - 1$, determine the value of $rmd(C_n^k)$.

Problem 2: For the power graph C_n^k with $n \geq 2k + 3$, determine the value of $u_{r_r}(C_n^k)$, $l_{r_r^*}(C_n^k)$, $u_{r_r^*}(C_n^k)$, $l_{R_r}(C_n^k)$, $u_{R_r}\left(C_n^k\right)$, $l_{R_r^*}(C_n^k)$, $u_{R_r^*}(C_n^k)$.

Acknowledgment

Authors are very much thankful to the learned referees for valuable suggestions, and the Principal and the Management of Dr. Ambedkar Institute of Technology, Bangalore for their constant support and encouragement during the preparation of this paper.

References

[1] F. Harary, *Graph theory*, Narosa Publishing House, New Delhi, 1969.

[2] F. Harary and R.A. Meltar, *On the metric dimension of a graph*, Ars Combin. **2**(1976), 191-195.

[3] G. Hartsfield and Ringel, *Pearls in Graph Theory*, Academic Press, USA, 1994.

[4] M. M. Padma and M. Jayalakshmi, *Boundary values of r_r, r_r^*, R_r, R_r^* sets of certain classes of Graphs*, Journal of Applied Mathematics and Bioinformatics, **6**(2) (2016), 143-154.

[5] M. M. Padma and M. Jayalakshmi, *On classes of rational resolving sets of derived graphs of a path*, Far East Journal of Mathematical Sciences, **110**(2) (2019), 247-259.

[6] P. J. Slater, *Dominating and references in a graph*, J. Math. Phys. Sci. **22**(1988), 445-455.

[7] A. Raghavendra, B. Sooryanarayana, C. Hegde, *Rational metric dimension of graphs*, Commun. Optim. Theory 2014 (2014), ID 8, 1-16.

[8] B. Sooryanarayana, *On the metric dimension of graph*, Indian J. Pure Appl. Math., **29**(4) (1998), 413-415.

[9] B. Sooryanarayana and B. Shanmukha, *A note on metric dimension*, Far East Journal of Mathematical Sciences, **5**(3)(2001), 331-339.

[10] G.K. Wong, D.A. Coppersmith, *A combinatorial problem related to multimodule memory organization*, J. Assoc. comput. Machin., **21**(1994), 392-401.