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Priority Retrial Queue with Re-Service and Working Breakdown Services

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Abstract:
The present investigation deals with single server priority retrial queue with re-service and working breakdown services. The Probability Generating Functions (PGF) for the system are found by using Supplementary Variable Technique (SVT). System performance measures and special cases are discussed.

Keywords:*priority queues; disasters; retrial queues; working breakdown services.*

I. INTRODUCTION

Queueing models with different service rates were studied by many authors [2]. The main motivation of these models is to change service rate depending on the situation of the system. Such as queues in random environment, working breakdown, models with vacations. Kalidass and Ramanath [4] have studied the concept of the working breakdowns. Recently, Kim and Lee [5] and Rajadurai [6] developed models in presence of Working Vacations (WV) and Working Breakdowns (WB).

In this work, retrial queue with priority arrivals in presence of disasters with WB services is introduced. In the period of WB, the server works in different rate of services. The model has convincing application in Wireless Sensor Networks (WSNs).

II. DESCRIPTION OF THE MODEL

A retrial queue with priority arrival in presence of re-service and disasters in WB services (M/G/1/WB priority policy) is addressed. In this model, we extend the work of Ammar and Rajadurai [1] by incorporating the concept of optional re-service.

Assume that two independent Poisson arrival rates are δ and λ for priority and ordinary customers. In optional re-service, as soon as the ordinary customer completes his service, he may repeat same service (without joining the orbit) with probability r or may leave the system with probability (1-r).

Considered as $R(0)=0$, $R(\infty)=1$, $S_p(0)=0$, $S_p(\infty)=1$, $S_w(0)=0$, $S_w(\infty)=1$, $S_b(0)=0$, $S_b(\infty)=1$ are continuous at $x = 0$ and $y = 0$.

Conditional completion rates for retrial, service on priority, ordinary and working breakdown respectively ($1 \le k \le m$),

$$\theta(x)dx = \frac{dR(x)}{1-R(x)}, \quad \mu_0(x)dx = \frac{dS_0(x)}{1-S_0(x)},$$

$$\mu_b(x)dx = \frac{dS_b(x)}{1-S_b(x)}, \; \mu_w(x)dx = \frac{dS_w(x)}{1-S_w(x)}.$$

III. STEADY STATE PROBABILITIES

"The steady state difference-differential equations and solutions are developed in this section.

$$\begin{aligned}(\lambda+\delta+\gamma)P_0 = \gamma P_0 &+ \int_0^\infty \Pi_{1,0}(x)\mu_p(x)dx + (1-r)\int_0^\infty \Pi_{b,0}(y)\mu_b(y)dy \\ &+ \int_0^\infty \Omega_{b,0}(y)\mu_b(y)dy + \int_0^\infty \Pi_{w,0}(x)\mu_v(x)dx + \alpha\int_0^\infty \Pi_{b,n}(y)dy \end{aligned} \tag{3.1}$$

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$$\frac{dP_n(x)}{dx}+\left(\lambda+\delta+\theta(x)\right)P_n(x)=0,\ n\geq 1 \quad (3.2)$$

$$\frac{d\Pi_{1,n}(x)}{dx}+\left(\lambda+\mu_p(x)\right)\Pi_{1,n}(x)=\lambda\Pi_{1,n-1}(x),\ n\geq 1 \quad (3.3)$$

$$\frac{\partial\Pi_{2,n}(x,y)}{\partial x}+\left(\lambda+\mu_p(x)\right)\Pi_{2,n}(x,y)=\lambda\Pi_{2,n-1}(x,y),\ n\geq 1, \quad (3.4)$$

$$\frac{d\Pi_{b,n}(y)}{dy}+\left(\lambda+\delta+\alpha+\mu_b(y)\right)\Pi_{b,n}(y)=\lambda\Pi_{b,n-1}(y)+\int_0^\infty \Pi_{2,n}(x,y)\mu_p(x)dx,\ n\geq 1 \quad (3.5)$$

$$\frac{d\Omega_{b,n}(y)}{dy}+\left(\lambda+\delta+\alpha+\mu_b(y)\right)\Omega_{b,n}(y)=\lambda\Omega_{b,n-1}(y)+\int_0^\infty \Pi_{2,n}(x,y)\mu_p(x)dx,\ n\geq 1 \quad (3.6)$$

$$\frac{d\Pi_{w,n}(x)}{dx}+\left(\lambda+\gamma+\mu_w(x)\right)\Pi_{w,n}(x)=\lambda\Pi_{w,n-1}(x),\ n\geq 1 \quad (3.7)$$

The steady state boundary conditions at $x = 0$ and $y = 0$ are

$$P_n(0)=\int_0^\infty \Pi_{1,n}(x)\mu_p(x)dx+(1\text{-}r)\int_0^\infty \Pi_{b,n}(y)\mu_b(y)dy+\int_0^\infty \Omega_{b,n}(y)\mu_b(y)dy+\int_0^\infty \Pi_{w,n}(x)\mu_w(x)dx,\ n\geq 1 \quad (3.8)$$

$$\Pi_{1,n}(0)=\delta\int_0^\infty P_n(x)dx,\ n\geq 1 \quad (3.9)$$

$$\Pi_{2,n}(0,y)=\delta\left(\Pi_{b,n}(y)+\Omega_{b,n}(y)\right),\ n\geq 0 \quad (3.10)$$

$$\Pi_{b,n}(0)=\left(\int_0^\infty P_{n+1}(x)a(x)dx+\lambda\int_0^\infty P_n(x)dx+\gamma\int_0^\infty \Pi_{w,n}(x)dx\right),\ n\geq 1 \quad (3.12)$$

$$\Omega_{b,n}(0)=r\int_0^\infty \Pi_{b,n}(y)\mu_b(y)dy,\ n\geq 1 \quad (3.13)$$

$$\Pi_{w,n}(0)=\begin{cases}(\lambda+\delta)P_0, & n=0.\\ 0, & n\geq 1.\end{cases} \quad (3.14)$$

The normalizing condition is

$$P_0+\sum_{n=1}^\infty\int_0^\infty P_n(x)dx+\sum_{n=0}^\infty\left(\int_0^\infty \Pi_{1,n}(x)dx+\int_0^\infty\int_0^\infty \Pi_{2,n}(x,y)dxdy\right)+\sum_{n=1}^{0}\left(\int_0^\infty \Pi_{b,n}(y)dy+\int_0^\infty \Omega_{b,n}(y)dy+\int_0^\infty \Pi_{w,n}(x)dx\right)=1 \quad (3.15)$$

Using the concept of SVT and PGF method, we get the steady state solutions of the model.

Theorem 1: The PGF's of number of customers in the orbit for different states,

$$P(z)=\frac{Nr(z)}{Dr(z)} \quad (3.16)$$

$$Nr=z(\lambda+\delta)\bar{R}^*(\lambda+\delta)P_0\left\{W(z)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)+\left(S_w^*\left(A_w(z)\right)-1\right)\right\}$$

$$Dr=\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)-z\delta\bar{R}^*(\lambda+\delta)S_p^*\left(A_p(z)\right)\right\}$$

$$\Pi_1(z)=\frac{\left(1-S_p^*\left(A_p(z)\right)\right)}{A_p(z)}\times\frac{Nr(z)}{Dr(z)} \quad (3.17)$$

$$Nr(z)=z\delta(\lambda+\delta)\bar{R}^*(\lambda+\delta)P_0\left\{W(z)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)+\left(S_w^*\left(A_w(z)\right)-1\right)\right\}$$

$$Dr(z)=\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)-z\delta\bar{R}^*(\lambda+\delta)S_p^*\left(A_p(z)\right)\right\}$$

$$\Pi_2(z)=\frac{\delta P_0\left(1+rS_b^*\left(A_b(z)\right)\right)\left(1-S_b^*\left(A_b(z)\right)\right)\left(1-S_p^*\left(A_p(z)\right)\right)}{\left(A_p(z)\times A_b(z)\right)}\times\frac{Nr(z)}{Dr(z)} \quad (3.18)$$

$$Nr=(\lambda+\delta)\left\{\left(S_w^*\left(A_w(z)\right)-1\right)\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)+zW(z)\left(1-\delta\bar{R}^*(\lambda+\delta)\right)\right\}$$

$$Dr=\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)-z\delta\bar{R}^*(\lambda+\delta)S_p^*\left(A_p(z)\right)\right\}$$

$$\Pi_b(z)=\frac{P_0\left(1-S_b^*\left(A_b(z)\right)\right)}{A_b(z)}\times\frac{Nr(z)}{Dr(z)} \quad (3.19)$$

$$Nr=(\lambda+\delta)\left\{\left(S_w^*\left(A_w(z)\right)-1\right)\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)+zW(z)\left(1-\delta\bar{R}^*(\lambda+\delta)\right)\right\}$$

$$Dr=\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)-z\delta\bar{R}^*(\lambda+\delta)S_p^*\left(A_p(z)\right)\right\}$$

$$\Omega_b(z)=\frac{rP_0S_b^*\left(A_b(z)\right)\left(1-S_b^*\left(A_b(z)\right)\right)}{A_b(z)}\times\frac{Nr(z)}{Dr(z)} \quad (3.20)$$

$$Nr(z)=(\lambda+\delta)\left\{\left(S_w^*\left(A_w(z)\right)-1\right)\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)+zW(z)\left(1-\delta\bar{R}^*(\lambda+\delta)\right)\right\}$$

$$Dr(z)=\left\{z-\left(R^*(\lambda+\delta)+\lambda z\bar{R}^*(\lambda+\delta)\right)\left(S_b^*\left(A_b(z)\right)(1-r)+r\left(S_b^*\left(A_b(z)\right)\right)^2+S(z)\right)-z\delta\bar{R}^*(\lambda+\delta)S_p^*\left(A_p(z)\right)\right\}$$

$$\Pi_w(z)=\left\{\frac{(\lambda+\delta)P_0W(z)}{\gamma}\right\} \quad (3.21)$$

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$$P_0 = \frac{R^*(\lambda+\delta)-\rho}{\eta} \qquad (3.22)$$

where

$$\rho = \left(R^*(\lambda+\delta) + \lambda\bar{R}^*(\lambda+\delta)\right)(1+r)\left(\lambda S_b^{*\prime}(\alpha)\left(1+\delta\beta_p^{(1)}\right) + S'(1)\right) + \delta\lambda\bar{R}^*(\lambda+\delta)\beta_p^{(1)}.$$

$$\eta = \left(R^*(\lambda+\delta)-\rho\right)\left(1+(\lambda+\delta)\left(1-S_w^*(\gamma)\right)/\gamma\right)$$
$$+\left(\lambda+\delta\right)\left\{1+\delta\beta_p^{(1)}\right\}\left(1-S_w^*(\gamma)\right)\left\{\begin{array}{l}\bar{R}^*(\lambda+\delta)\left(\dfrac{\lambda}{\gamma}+\lambda S_b^{*\prime}(\alpha)\left(1+\delta\beta_p^{(1)}\right)+S'(1)\right)+ \\ \bar{S}_b^*(\alpha)\left\{\begin{array}{l} R^*(\lambda+\delta)-\delta\lambda\bar{R}^*(\lambda+\delta)\beta_p^{(1)} \\ +\dfrac{\lambda}{\gamma}\left(1-\delta\bar{R}^*(\lambda+\delta)\right)\end{array}\right\}\end{array}\right\}$$

$$S'(1) = \frac{\lambda\left(1+\delta\beta_p^{(1)}\right)}{\alpha}\left(1-S_b^*(\alpha)+\alpha S_b^{*\prime}(\alpha)\right); \quad \bar{S}_b^*(\alpha) = \left(\frac{1-S_b^*(\alpha)}{\alpha}\right);$$

$$W'(1) = \frac{\lambda}{\gamma}\left(1-S_w^*(\gamma)+\gamma S_b^{*\prime}(\alpha)\right); \quad A_b' = \lambda\left(1+\delta\beta_p^{(1)}\right);$$

Proof: Solving (3.2)-(3.14) we get the define the partial PGFs and integrate w.r.to 'x' & 'y'

$$P(z) = \int_0^\infty P(x,z)dx; \;\; \Pi_1(z) = \int_0^\infty \Pi_1(x,z)dx; \;\; \Pi_2(z) = \int_0^\infty \Pi_2(x,z)dx;$$

$$\Pi_b(z) = \int_0^\infty \Pi_b(y,z)dy; \;\; \Omega_b(z) = \int_0^\infty \Omega_b(y,z)dy; \;\; \Pi_w(z) = \int_0^\infty \Pi_w(x,z)dx;$$

Using the normalizing condition, we get the idle probability.

Corollary 2: *The system in stability condition* $\rho < R^*(\lambda+\delta)$,

The PGF of system size ($K_s(z)$) is

$$K_s(z) = P_0 + P(z) + z\left(\Pi_1(z)+\Pi_2(z)+\Pi_b(z)+\Omega_b(z)+\Pi_w(z)\right).$$

The PGF of orbit size ($K_o(z)$) is

$$K_o(z) = P_0 + P(z) + \Pi_1(z) + \Pi_2(z) + \Pi_b(z) + \Omega_b(z) + \Pi_w(z).$$

IV. Performance Measures

From Eqns. (3.16) - (3.21), then we get the different steady state probabilities,

$$P = P(1);\ \Pi_1 = \Pi_1(1);\ \Pi_b = \Pi_b(1);\ \Pi_2 = \Pi_2(1);\ \Omega_b = \Omega_b(1);\ \Pi_w = \Pi_w(1).$$

The mean orbit size (L_q) is $L_q = K_o'(1) = \lim_{z\to 1}\frac{d}{dz}K_o(z)$

The mean system size (L_s) is $L_s = K_s'(1) = \lim_{z\to 1}\frac{d}{dz}K_s(z)$

The average time a customer spends in the system (W_s) and queue (W_q)" $W_s = L_s/\lambda$ and $W_q = L_q/\lambda$.

V. Special Cases

Case 1: Here, we put $\gamma = \alpha = 0$, results are equivalent to retrial queues with priority arrivals by Gao [3].

Case 2: $R^*(\lambda)\to 1$ and $\delta = 0$, results are equivalent by Kalidass and Ramanath [4]

VI. Conclusion

A retrial queue with priority arrivals in presence of re-service and disasters in WB services (M/G/1/WB priority policy) has been discussed in this work. Using the method of SVT, the system size and its orbit are found. System characteristics like steady state probabilities and the mean system size are found. Practical application of this model is in Wireless Sensor Networks (WSNs) in PRIN MAC protocol.

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