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An Unreliable Retrial Queue with Instant Feedback and Working Vacation

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Abstract:
In this work, a retrial queue with single server is addressed with different performance measure parameters, such as immediate respond feedback, working vacation and orbit search. The Probability Generating Function (PGF) of the retrial queuing system with various orbit size is studied by using Supplementary Variable Technique (SVT). The mathematical model is developed to measure mean system size and orbit size the under certain conditions. Special cases with suitable arrival and departure patterns are analysed.

Keywords:*retrial queue; immediate feedback; orbit search; working vacation.*

I. INTRODUCTION

In the literature of retrial queueing system, it is presented that retrial queues associated with various kinds of customers have been investigated by many authors who have applications in analyzing the performance of communication networks, traffic management, etc [4]. For a detailed review on retrial queues refer the survey papers of Artalejo [1]. The idea of immediate feedback service in a retrial queue is presented by Kalidass and Kasturi [5] which has been developed with inclusion of breakdown and vacation queues by Varalakshmi et al. [7].

In the past, literatures on queues are associated with static service rates. Recently, queueing models with difference in service rates due to interruptions on vacations and Working Vacation (WV), breakdowns or the environment were investigated by many authors. Retrial queues with WV concept are analysed by Gao et al. [3].

Mostly, in queuing literature assumes that, at the service completion moment of the server waits idle for the arrival of next customer (primary or retrial) in the system. In reality, we (source/ manager) want to minimize the holding cost and the server's idle time. Thus study on the search of customer from the orbit in a retrial queue becomes prominent. That is, at the end of each service, the server can search for customer, if the search is successful; the pattern of service is followed by another service. Else, it is followed by idle period. For recent research on retrial queues with orbital search refer Gao and Wang [2] and Rajadurai et al. [6]. This motivates the researchers to study an unreliable single server retrial queue single working vacation policy with immediate feedback with search of customers from the orbit.

II. DESCRIPTION OF THE MODEL

A retrial queueing system with orbit search, instant feedback under working vacation policy is considered in this paper.

According to the general rule of retrial, service, working vacation and repair process are considered as general distribution (Varalakshmi et al. [7]). Where λ is the Poisson arrival rate, β is the breakdown rate, p is the vacation rate, $\alpha_i$is the feedback probability of i^{th} phase, LST for retrial $R^*(\lambda)$, busy$S_b^*(v)$, repair $G^*(v)$ and WV $S_v^*(v)$. At the end of a vacation, the server finds for the customers in the orbit with probability θ or remains idle with probability$(1-\theta)$.

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Assume that
$R(0)=0, R(\infty)=1, S_b(0)=0, S_b(\infty)=1,\ G_j(0)=0,\ G_j(\infty)=1, S_v(0)=0,\ S_v(\infty)=1,$
are continuous at x = 0 and y = 0. Hazard rates for different states (for j=0,1, 2,…,m-1).

$$a(x)dx=\frac{dR(x)}{1-R(x)},\ \mu_b(x)dx=\frac{dS_b(x)}{1-S_b(x)},\ \mu_v(x)dx=\frac{dS_v(x)}{1-S_v(x)},\ \xi_j(y)dy\ =\frac{dG_j(y)}{1-G_j(y)}.$$

The system is ergodic for the embedded Markov chain $\{Z_n;\ n\in N\}$ if and only if $\rho<1$ and for our system represented in this paper is stable with suitable conditions, where $\rho=1-\Phi'(1)+\bar{\theta}\left(1-R^*(\lambda)\right)$

III. STEADY STATE PROBABILITIES

The steady state equations and solutions are developed in this section.

3.1.The steady state equations

"By the method of SVT, we obtain the following equations (0≤j≤ m-1).

$$\lambda P_0\ = pQ_0 \qquad (1)$$

$$(\lambda+p)Q_0\ =(1-\theta)\sum_{l=0}^{m-1}\bar{\alpha}_{l+1}\int_0^\infty Q_{l,0}(x)\mu_b(x)dx+\int_0^\infty\Omega_{v,0}(x)\mu_v(x)dx;\alpha_m=0 \qquad (2)$$

$$\frac{d\psi_n(x)}{dx}+\left(\lambda+a(x)\right)\psi_n(x)\ =\ 0,\ n\geq 1 \qquad (3)$$

$$\frac{dQ_{j,n}(x)}{dx}+\left(\lambda+\mu_b(x)+\beta\right)Q_{j,n}(x)\ =\ \lambda Q_{j,n-1}(x)+\int_0^\infty R_{j,n}(x,y)\xi_i(y)dy \qquad (4)$$

$$\frac{d\Omega_{v,n}(x)}{dx}+\left(\lambda+\mu_v(x)+p\right)\Omega_{v,n}(x)\ =\ \lambda\Omega_{v,n-1}(x) \qquad (5)$$

$$\frac{dR_{j,n}(x,y)}{dy}+\left(\lambda+\xi_i(y)\right)R_{j,n}(x,y)\ =\ \lambda R_{j,n-1}(x,y);\ n\geq 1 \qquad (6)$$

The steady state boundary conditions at x = 0 are

$$\psi_n(0)\ =(1-\theta)\sum_{l=0}^{m-1}\bar{\alpha}_{l+1}\int_0^\infty Q_{l,n}(x)\mu_b(x)dx+\int_0^\infty\Omega_{v,n}(x)\mu_v(x)dx, \qquad (7)$$

$$Q_{0,n}(0)\ =\int_0^\infty\psi_{n+1}(x)a(x)dx+\lambda\int_0^\infty\psi_n(x)dx+p\int_0^\infty\Omega_{v,n}(x)dx+\theta\sum_{l=0}^{m-1}\bar{\alpha}_{l+1}\int_0^\infty Q_{l,n+1}(x)\mu_b(x)dx+\lambda P_0 \qquad (8)$$

$$Q_{j,n}(0)\ =\alpha_j\int_0^\infty Q_{j-1,n}(x)\mu_b(x)dx,\ j=1,2,3,...,m-1 \qquad (9)$$

$$\Omega_{v,n}(0)\ =\begin{Bmatrix}\lambda Q_0, n=0\\ 0, n\geq 1\end{Bmatrix} \qquad (10)$$

$$R_{j,n}(x,0)\ =\beta Q_{j,n}(x)dx; n\geq 0 \qquad (11)$$

The normalizing condition is

$$Q_0+P_0+\sum_{n=1}^{\infty}\int_0^\infty\psi_n(x)dx+\sum_{n=0}^{\infty}\sum_{j=0}^{m-1}\left(\int_0^\infty Q_{j,n}(x)dx+\int_0^\infty\int_0^\infty R_{j,n}(x,y)dxdy\right)+\sum_{n=0}^{\infty}\int_0^\infty\Omega_{v,n}(x)dx=1 \qquad (12)$$

(13)

3.2. The Steady State Solutions

The PGF for the different state are,

$$\psi(x,z)=\sum_{n=1}^{\infty}\psi_n(x)z^n\ ;\ Q_j(x,z)=\sum_{n=0}^{\infty}Q_{j,n}(x)z^n;$$

$$\Omega_v(0,z)=\sum_{n=0}^{\infty}\Omega_{v,n}(0)z^n\ ; R_j(x,y,z)=\sum_{n=0}^{\infty}R_{j,n}(x,y)z^n\ ;$$

From eqns. (1) to (11) by $z^n$and take summation n, (n = 0,1,2...,∞) (for j=0,1,…,m-1) and solving the PDE, then we get the following limiting PGF $\psi(x,z), Q_j(x,z), \Omega_v(x,z), R_j(x,z).$

Results: If the stability condition ρ <1,
The PGF of the number of customers in the orbit for different states are

$$\psi(z)\ =\int_0^\infty\psi(x,z)dx=\frac{zQ_0\left(1-R^*(\lambda)\right)}{\lambda Dr(z)}\begin{Bmatrix}z\left(p+\lambda V(z)\right)\bar{\theta}\Phi(z)\\ +\left(\lambda\left(S_v^*\left(A_v(z)\right)-1\right)-p\right)\left(z-\Phi(z)\right)\end{Bmatrix} \qquad (13)$$

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$$Q_{j_b}(z)=\int_0^{\infty}Q_{j_b}(x,z)dx \quad (14)$$

$$=Q_0\left(\prod_{j=0}^{m-1}\alpha_j\right)\left(S_b^*\left(A_b(z)\right)\right)^j\left(1-S_b^*\left(A_b(z)\right)\right)$$

$$\times\frac{\left[z\left(p+\lambda V(z)\right)+R(z)\left(\lambda\left(S_v^*\left(A_v(z)\right)-1\right)-p\right)\right]}{Dr(z)\times A_b(z)}$$

$$\Omega_v(z)=\int_0^{\infty}\Omega_v(x,z)dx=\frac{\lambda Q_0 V(z)}{p} \quad (15)$$

$$R_j(z)=\int_0^{\infty}P_{j_b}(x,z)dx=\beta Q_0\left(\prod_{j=0}^{m-1}\alpha_j\right)\left(S_b^*\left(A_b(z)\right)\right)^j\left(1-S_b^*\left(A_b(z)\right)\right) \quad (16)$$

$$\times\left(1-G_j^*\left(b(z)\right)\right)\frac{\left[z\left(p+\lambda V(z)\right)+R(z)\left(\lambda\left(S_v^*\left(A_v(z)\right)-1\right)-p\right)\right]}{Dr(z)A_b(z)b(z)}$$

By using the following result,

$$P_0+Q_0+\psi(1)+\Omega_v(1)+\sum_{j=0}^{m-1}\left(Q_j(1)+R_j(1)\right)=1.$$

We get $Q_0=\dfrac{1-\rho}{\omega+\Gamma}$ (17)

$$\Gamma=\sum_{j-0}^{m-1}\left(\prod_{j=0}^{m-1}\alpha_j\right)\lambda E(S_b)\left(1+\beta E(G_j)\right)\left[\frac{\lambda}{p}V(z)-R^*(\lambda)\left(S_v^*(p)-\left(1+\frac{p}{\lambda}\right)\right)\right]$$

$$Dr(z)=z-\Phi(z)\left(\theta+\bar{\theta}R(z)\right);\; V(z)=p\left(1-S_v^*(A_v(z))\right)/A_v(z)$$

$$A_b(z)=b(z)+\beta[1-G_j^*(b(z))],A_v(z)=b(z)+p,b(z)=\lambda(1-z)$$

$$\Phi(z)=\sum_{l=0}^{m-1}\bar{\alpha}_{l+1}\left(\prod_{l=0}^{m-1}\alpha_l\right)\left(S_b^*(A_b(z))\right)^{l+1}$$

$$R(z)=R^*(\lambda)+z\left(1-R^*(\lambda)\right);\; \rho=\Phi'(1)+\bar{\theta}\left(1-R^*(\lambda)\right)$$

$$\omega=\left(1-\Phi'(1)\right)\left(\left(1+\frac{p}{\lambda}\right)R^*(\lambda)+\frac{\lambda}{p}V(1)\right)-\left(1-R^*(\lambda)\right)S_v^*(p)\left(\bar{\theta}+\Phi'(1)\right)$$

$$V(1)=1-S_v^*(p)$$

The PGF of number of customers in the system ($K_s(z)$) and in the orbit ($K_o(z)$) size distribution at stationary point of time is

$$K_s(z)=Q_0+P_0+\psi(z)+\Omega_v(z)+\sum_{j=0}^{m-1}z\left(Q_j(z)+R_j(z)\right) \quad (19)$$

$$K_o(z)=Q_0+P_0+\psi(z)+\Omega_v(z)+\sum_{j=0}^{m-1}\left(Q_j(z)+R_j(z)\right) \quad (20)$$

IV. PERFORMANCE MEASURES

(i) The mean orbit siz (L_q) is, $L_q=\lim_{z\to 1}\frac{d}{dz}K_o(z)$

(ii) The mean system size (L_s) is, $L_s=\lim_{z\to 1}\frac{d}{dz}K_s(z)$

(iii) The mean waiting time of system (W_s) and the orbit (W_q) as", $W_s=\frac{L_s}{\lambda}$ and $W_q=\frac{L_q}{\lambda}$

V. SPECIAL CASES

Case (i): Let $R^*(\lambda)\to 1;\alpha_j=\theta=\beta=0$; then the model is reduced to M/G/1 queue with working vacation policy.

Case (ii): Let $\alpha_j=\theta=\mu_v=\beta=0$; then this model reduced to M/G/1 retrial queue with general retrial times.

VI. CONCLUSION

In this manuscript, we discussed a single server retrial queuing system with single working vacation, immediate feedbacks and orbital search where the single server is subject to failure and repair. With different customer size, the state of steady state probability generating function is monitored by the use of SVT. Various performance measures special cases are deduced. This model is useful to find in transmission control protocols and wireless network applications.

$$(17)\; P_0=\frac{p/}{\omega+\sum_{j-0}^{m-1}\left(\prod_{j=0}^{m-1}\alpha_j\right)\lambda E(S_b)\left(1+\beta E(G\right.}$$

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