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Probability of Lot Acceptance for Two-Sided Chain Sampling Strategy based on Exponential Distribution

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Abstract:This article suggests two-sided chain sampling strategy for exponential distribution. The proposed plan has two acceptance criteria, where the first criterion is the lot is accepted when the current lot contains no nonconforming units. For the second criterion, the lot is accepted when there is one nonconforming unit found provided that the cumulative number of nonconforming units from the preceding and succeeding lots is at most one. Otherwise, the lot is rejected. For the proposed plan, the performance is measured based on the probability of lot acceptance, P_a (p), where it is calculated at various values of mean ratio. The proposed plan eventually will provide an alternative plan for the industrial practitioners to choose from for the inspection of a product.

Keywords: *Probability of lot acceptance, Consumer's risk, Exponential distribution, Two-sided chain sampling strategy.*

I. INTRODUCTION

The pioneers in acceptance sampling were actually Dodge, Miller and others. However, Dodge and his colleagues did not come out with anything, instead Dodge partnered with Romig to produce a table called Sampling Inspection Tables. The table was basically generated by using two points which are (i) the lot tolerance percent defective (LTPD) and, (ii) the average outgoing quality level (AOQL).LTPD is the least quality of a product that a customer is agreeable to accept while the AOQL is the poorestlikely average quality from the correcting inspection activity [1].

The above scenario is related with Statistical Quality Control (SQC), where SQC involves two main areas. The first area is the process control and the second area is acceptance sampling. Process controls deal with charts producing where the charts usually have control limits (lower and upper) while acceptance sampling concerns about the product inspection and the final decision in the acceptance sampling is always either to take or to discard the lot. In this article, it focuses on the acceptance sampling, particularly the two-sided chain sampling strategy.

The acceptance sampling has developed so much over the years. Many researchers have studied the acceptance sampling rigorously such as [2]-[25]. Therefore, this article will investigate the proposed

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plan by considering exponential distribution at different design parameters. The plan is chosen as it improves the two-sided modified ChSP-1(TSMChSP-1) developed by [4] and exponential distribution is selected as many electric and electronic equipment may exhibit the exponential distribution.

II. GLOSSARY OF SYMBOLS

There are lots of symbols used throughout the article. Forthe convenience purpose, all the symbols are described here:

c : Acceptance number
n : Sample size
d : Number of units (nonconforming) in the lot
d_i : Number of units (nonconforming) in the i lots
d_j : Number of units (nonconforming) in the j lots
i : Number of preceding lots
j : Number of succeeding lots
P_0 : Probability of zerounit (nonconforming)
P_1 : Probability of one unit (nonconforming)
a : Specified constant
p : Fraction defective
σ : Scale parameter
β : Consumer's risk
t_0 : Inspection time
$P_a(p)$: Probability of lot acceptance
μ : True life (mean)
μ_0 : Indicated life (mean)
D : Cumulative number of units (nonconforming)
$\frac{\mu}{\mu_0}$: Mean ratio

III. LITERATURE REVIEW

Single acceptance sampling plans (SSP) initiated by Epstein [2], is the most basic sampling plan among other established acceptance sampling plans. The steps of SSP, based on Montgomery [1]is to select and inspect one sample from the submitted lot, and the decision whether to take or discard the lot is based on the inspection result of the sample taken.

Goode and Kao [15] suggested the extended of SSP and the introduction of the reliability sampling plan. For that study, Weibull distribution is used as a lifetime distribution to examine the mean lifetime of a submitted product. Their plan is the continuation of established sampling plan developed by Epstein [2]. It is very helpful for experimenters, when their problem related to Military Standard 105B and reliability purpose.

Gupta and Groll [33] realized that different distribution has different fraction defective, therefore the studied the plan for Gamma distribution for the SSP. They kept the steps for the SSP as suggested by Epstein [2], but they only made the difference in the fraction defective. Tables were generated to illustrate the n and $P_a(p)$ at different design parameters.

Dodge [3] proposed the chain acceptance sampling plan (ChSP-1) to unravel the issue in the SSP,where the SSPhave problem when the $c = 0$ or $c = 1$.For the ChSP-1, the criteria to accept the sampling plan do not rely on one lot only, but it depends on the cumulative knowledge from the i lots.This sampling plan provides another chances for the lot to be accepted if one nonconforming unit is found, given that there are no nonconforming units found in the i lots compared to the SSP.

Ramaswamy and Jayasri [28] developed ChSP-1 for generalized Rayleigh distribution. The n isfound when it satisfies different design parameters. Apart from generalized Rayleigh distribution, they also studied the plan for other distributions such as generalized exponential distribution [30], log-logistic distribution [31] and inverse Rayleigh distribution [32]. Ramaswamy and Jayasri [29] also presented the modified ChSP-1 considering several lifetime distributions.

Govindaraju and Lai [15] improved the ChSP-1 by modifying the acceptance criteria in the plan and renamed it as modified ChSP-1 (MGChSP-1). The acceptance criteria now is the lot is accepted if there is no nonconforming units found given that there is at most one nonconforming unit found in the i lots. Other than this scenario, the lot is rejected.

Vijila and Deva [26] realized that the plans developed by [3] and [15] only acknowledged the i lots and overlooked at the j lots. Therefore, they proposed a new acceptance sampling plan that acknowledged both, i and j lots and called it as two-sided complete ChSP-1 (TSCChSP-1). They

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listed out the operating procedures for the plan and concluded the plan that increasing the preceding lots made the sample size converged to a constant.

IV. METHODOLOGY

For the suggested plans, the operating steps are listed below:

i. The n is found by solving $P_a(p) \leq \beta$.
ii. The t_0 is specified.
iii. The d is counted during thet_0.
iv. The lot is accepted if $d = 0$ given that$d_i = 0$ and $d_j = 0$. Reject the lot if $d > 1$.
v. The lot is also accepted if $d = 1$ given that $D = d_i + d_j = 1$).

V. PROBABILITY OF LOT ACCEPTANCE

The $P_a(p)$ for the suggested plan is

$$P_a(p) = (P_0)^{2i+1} + 2iP_1(P_0)^{2i}. \quad (1)$$

The P_0 and P_1 are derived by using the binomial distribution, $P_d(p)$, where$P_d(p)$ is

$$P_d(p) = \binom{n}{d}(p)^d(1-p)^{n-d}. \quad (2)$$

For the TSChSP-1, there are two conditions where the lot is accepted. The first condition is when the current lot has no nonconforming units while the second condition is the current lot has one nonconforming unit given that the $D = d_i + d_j = 1$. By substituting $d = 0$ and $d = 1$ in (2), then (2) can be rewritten as

$$P_0(p) = (1-p)^n \quad (3)$$

and

$$P_1(p) = np\,(1-p)^{n-1}. \quad (4)$$

Then, (3) and (4) are substituted in (1), and then (1) can be printed as

$$P_a(p) = (1-p)^{n(2i+1)}\left[1 + \frac{2inp}{1-p}\right]. \quad (5)$$

VI. EXPONENTIAL DISTRIBUTION

In this article, the fraction defective, p is derived by using the cumulative distribution function (CDF) of exponential distribution, and it is printed as

$$F(t;\,\sigma) = 1 - exp\left(-\frac{t}{\sigma}\right), \quad t > 0. \quad (6)$$

The mean is

$$\mu = \sigma. \quad (7)$$

Thet_0 is written as

$$t_0 = a\mu_0. \quad (8)$$

The$p$is

$$p = 1 - exp\left[-a\left(\frac{\mu_0}{\mu}\right)\right]. \quad (9)$$

VII. RESULT AND DISCUSSION

In this study, the design parameters used are $\beta = 0.10, 0.05, 0.01$; $a = 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00$; $i = 1, 2, 3, 4, 5$ and $j = 1, 2, 3, 4, 5$.

Table I shows the $P_a(p)$ for exponential distribution.

Table I: The$P_a(p)$ for exponential distribution

β	n	a	$\frac{\mu}{\mu_0}$						
			1	2	4	6	8	10	12
0.01	9	0.25	0.00716	0.11623	0.39973	0.57329	0.67584	0.74116	0.78569
	5	0.50	0.00414	0.09031	0.35755	0.53549	0.64417	0.71455	0.76300
	3	0.75	0.00902	0.12763	0.41387	0.58401	0.68373	0.74709	0.79027
	3	1.00	0.00140	0.05435	0.28502	0.46593	0.58401	0.66313	0.71867
	2	1.25	0.00606	0.10519	0.37838	0.55195	0.65652	0.72395	0.77032

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	2	1.50	0.00184	0.06074	0.29722	0.47663	0.59247	0.66976	0.72395
	2	1.75	0.00055	0.03461	0.23147	0.40917	0.53239	0.61764	0.67866
	2	2.00	0.00016	0.01952	0.17898	0.34950	0.47663	0.56794	0.63476
0.05	6	0.25	0.04897	0.27380	0.57591	0.71354	0.78683	0.83133	0.86091
	4	0.50	0.01534	0.16291	0.46081	0.62364	0.71609	0.77392	0.81298
	3	0.75	0.00902	0.12763	0.41387	0.58401	0.68373	0.74709	0.79027
	2	1.00	0.01952	0.17898	0.47663	0.63475	0.72395	0.77969	0.81737
	2	1.25	0.00606	0.10519	0.37838	0.55195	0.65652	0.72395	0.77032
	2	1.50	0.00184	0.06074	0.29722	0.47663	0.59247	0.66976	0.72395
	2	1.75	0.00055	0.03461	0.23147	0.40917	0.53239	0.61763	0.67866
	1	2.00	0.03415	0.22088	0.51263	0.65895	0.74069	0.79183	0.82653
0.10	5	0.25	**0.09031**	0.35754	0.64417	0.76300	0.82443	0.86128	**0.88563**
	3	0.50	0.05435	0.28502	0.58401	0.71867	0.79027	0.83378	0.86274
	2	0.75	0.06074	0.29722	0.59247	0.72395	0.79379	0.83627	0.86459
	2	1.00	0.01952	0.17898	0.47663	0.63475	0.72395	0.77969	0.81737
	2	1.25	0.00606	0.10519	0.37838	0.55195	0.65652	0.72395	0.77032
	1	1.50	0.08847	0.34086	0.62008	0.74069	0.80480	0.84401	0.87031
	1	1.75	0.05515	0.27511	0.56458	0.69921	0.77250	0.81782	0.84839
	1	2.00	0.03415	0.22088	0.51263	0.65895	0.74069	0.79183	0.82653

Based on the Table I, the$P_a(p)$rises as the mean ratio, $\frac{\mu}{\mu_0}$rises. For illustration, if the design parameter are $\left(\beta, n, a, i, \frac{\mu}{\mu_0}\right) = (0.1, 5, 0.25, 1, 1)$ then the $P_a(p)$ is 0.09031. The $P_a(p)$rises from 0.09031 to 0.88563 when the mean ratio, $\frac{\mu}{\mu_0}$rises from 1 to 12, keeping the other design parametersunbothered. It indicates that the $P_a(p)$rises from 9.03% to 88.56% when the actual average life of a unit is higher (12 times higher) than the stated mean life. In the other word, when the producer produces a higher quality product which indicated by higher mean ratio, it is expected that the $P_a(p)$ would be higher.

VIII. CONCLUSION

This article proposed the TSChSP-1 for exponential distribution. The TSChSP-1 is an improvement of the TSMChSP-1 as it has higher lot acceptance criteria compared to the TSMChSP-1. By implementing the TSChSP-1 for exponential distribution, industrial practitioners now have more options of sampling plans to choose from for their inspection activity.

The two sampling plans have been studied for exponential distribution only. With many lifetime distributions in the literature, it is worthwhile to study the plans with other lifetime distributions as different products may actually have different lifetime distributions.

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