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Relationship between the Neighborhood Polynomial and Majority Neighborhood Polynomial of a Graph

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Abstract
In this article we determine the relationship between the neighborhood polynomial $N(G,x)$ and majority neighborhood polynomial $N_M(G,x)$ andt their coefficients. Also found some bounds of these polynomials of graphs and coeffiecients of polynomials.

Index Terms: *Neighborhood number, neighborhood polynomial, Majority neighborhood number and majority neighborhood polynomial Number*

1. Introduction

The relationship between the neighborhood polynomial $N(G,x)$ and the majority neighborhood polynomial of a graph $N_M(G,x)$ are parameters studied in detail in these article [11] - [17],.

A set $S \subseteq V(G)$ is called a majority neighborhood set if $G_M = \bigcup_{u \in S} \langle N[u] \rangle$ contains at least $\left\lceil \frac{p}{2} \right\rceil$ vertices and at least $\left\lceil \frac{q}{2} \right\rceil$ edges. A majority set S is called a minimal majority neighborhood set if no proper subset of S is a majority neighborhood set. The minimum cardinality of a majority neighborhood set is called the majority neighborhood number of G and is denoted by $N_M(G)$ [9],[24]-[26].

The neighborhood polynomial $N(G,x)$ of G is defined in as $N(G,x) = \sum_{i=n(G)}^{n} N(G,i)x^i$ where $N(G,i)$ is family of neighborhood sets with cardinality i. The majority neighborhood polynomial of G is defined as $N_M(G,x) = \sum_{i=n_M(G)}^{p} N_M(G,i)x^i$. The $N_M(G,i)$ is family of neighborhood sets with cardinality i and $n_o(G,i) = |N(G,i)|$. In this article we found some relationships between the coefficient and polynomials.

The majority neighborhood number has been studied by Joseline Manora and Swaminathan in [9].

2. Relationship Between $N(G,x)$ and $N_M(G,x)$.

Theorem 2.1. Let G be a connected graph with p vertices. If all the vertices of G are full degree then the neighborhood polynomial and majority neighborhood polynomials are the same**.**

Proof. Since all the vertices of a graph G are full degree, the induced subgraph of each vertex covers all vertices and edges. Therefore, $n_o(G) = n_M(G) = 1$. From the definition of $N(G,x)$ and $N_M(G,x)$ one can construct these polynomials with size starting from $i = 1,2,3,...p$. Therefore in both polynomials we get the same number of possible neighborhood set and majority neighborhood sets of sizes i = 1, 2,3,…,p for G. Thus we get the neighborhood polynomial which is same as the majority neighborhood polynomial for G.

Example. For the complete graph $K_{1,5}$ the $n_0(G) = n_M(G) = 1$ then $N(G,x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x^5 = N_M(G,x)$

Theorem 2.2. Let G be a disconnected graph with m components such that $G = \bigcup_{i=1}^{m} G_i$. Then the structure of neighborhood polynomial and that of majority neighborhood polynomial are the same. But the

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polynomials are not same.

Proof. Let $G=\bigcup_{i=1}^{m} G_i$. For each component G_i, $i=1,2,3,...,m$. The neighborhood polynomial and majority neighborhood polynomial are obtained independently then take the product of these polynomials of, G_i, $i=1,2,3,...m$, we get the neighborhood polynomial $N(G,x)$ of G. Similarly, we get $N_M(G,x)$, majority neighborhood polynomial of G. Therefore, $N(G,x)=\prod_{i=1}^{m} N(G_i,x)$ and $N_M(G,x)=\prod_{i=1}^{m} N_M(G_i,x)$. Hence these two polynomials have the same structure. At the same time, for each components G_i, $i=1,2,3,...,m$, neighborhood number $n_o(G)$ and majority neighborhood number $n_M(G)$ are different. Therefore the coefficients of these two polynomials are different.

Theorem 2.3. Let G be a totally disconnected graph with p vertices. Then the neighborhood polynomial $N(G,x)$ and majority neighborhood polynomial $N_M(G,x)$ are different.

Proof. Let $G=\overline{K_p}, p\geq 2$. Since the neighborhood number $n_0(G)=p$ and $n_M(G)=\left\lceil \frac{p}{2} \right\rceil$, $N(G,x)$ contains only one set with size $i=p$. Therefore, $N(G,x)=x^p$. But $N_M(G,x)=\sum_{i=\left\lceil \frac{p}{2} \right\rceil}^{p} N_M(G,i)x^i=\sum_{i=\left\lceil \frac{p}{2} \right\rceil}^{p} \binom{p}{i} x^i$. Thus the neighborhood polynomial and majority neighborhood polynomial of a graph G is different.

Theorem 2.4. If a connected graph G has exactly one full degree vertex then the neighborhood polynomial and majority neighborhood polynomial are different.

Proof. Let G be any connected graph with a full degree vertex u. Therefore, the neighborhood number $n_0(G)=1$ and also majority neighborhood number $n_M(G)=1$. From the definition of neighborhood polynomial, both the polynomials are defined by $N(G,x)=\sum_{i=1}^{p} N(G,i)x^i$ and $N_M(G,x)=\sum_{i=1}^{p} n_M(G,i)x^i$. Since every neighborhood set must cover all the vertices and edges of G, there are neighborhood sets with different sizes along with a full degree vertex u only. In some cases, there is exactly one combination of neighborhood sets excluding u and in other cases, there are few possible neighborhood sets excluding the full degree vertex u. The structure of neighborhood polynomial is

$$\binom{p}{0}x+\binom{p}{1}x^2+\binom{p}{1}x^3+...+\binom{p}{p}x^{p+1}+fewterms;$$

.

In the case of majority neighborhood set, each one must cover at least $\left\lceil \frac{p}{2} \right\rceil$ vertices and at least $\left\lceil \frac{q}{2} \right\rceil$ edges. Here we get the same number of majority neighborhood sets of sizes $i=1,2,3,...,p$ including full degree vertex plus there are many possible combination of majority neighborhood sets of different sizes excluding the full degree vertex u Therefore the structure of majority neighborhood polynomial is

$$N_M(G,x)=\binom{p}{0}x+\binom{p}{1}x^2+\binom{p}{1}x^3+...+\binom{p}{p}x^{p+1}+moreterms;$$

Hence both neighborhood and majority neighborhood polynomials are different.

Theorem 2.5. If a graph G has no full degree vertex then the coefficients of the neighborhood polynomial less than or equal to the coefficients of majority neighborhood polynomial and also neighborhood polynomial contains no 'x' terms.

Proof. Since G has no full degree vertex, $n_0(G)>1$ but $n_M(G)\geq 1$. Then the neighborhood polynomial becomes $N(G,x)=\sum_{i=2}^{p} N(G,i)x^i$ but majority neighborhood polynomial becomes $N_M(G,x)=\sum_{i=1}^{p} n_M(G,i)x^i$.

In neighborhood sets, each set covers p vertices and q edges. But in the case of majority neighborhood sets, each set covers at least $\left\lceil \frac{p}{2} \right\rceil$ vertices and $\left\lceil \frac{q}{2} \right\rceil$ edges. Therefore the total number of neighborhood sets of different sizes $i=1,2,3,...,p$ is lesser than the total number of majority neighborhood sets of different sizes $i=1,2,3,...,p$ .Certainly, the coefficients of neighborhood polynomial less than or equal to the coefficients of majority neighborhood polynomial of G. Since $n_0(G)\geq 2$ and G has no full degree vertex, there is no neighborhood sets with size $i=1$. Hence the neighborhood polynomial contains no ‘ x ’ terms.

Theorem 2.6. For both neighborhood and majority neighborhood polynomial of a graph G, the coefficient of $x^p=1$.

Proof. Since the whole vertex set is a neighborhood set as well as a majority neighborhood set of a graph G, the coefficient of x^p is equal to 1.

3. Conclusion

The researcher found the relationship between the neighborhood polynomials and majority neighborhood polynomials and coefficients. Further proceed to determine the relationship between independent, connected and majority polynomial of these parameters.

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