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Complement Connected Majority Neighborhood Number

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Abstract

A set of vertices S in a graph G is a majority neighborhood set of G if the subgraph $\cup_{v\in S}\langle N[v]\rangle$ contains atleast $\left\lceil\frac{p}{2}\right\rceil$ vertices and $\left\lceil\frac{q}{2}\right\rceil$ edges, where $\langle N[v]\rangle$ is the subgraph of G. In this article majority neighborhood connected complement set of G are introduced and this number determined $n_{\overline{CCM}}(G)$ for various graph structures.

Keywords: *Connected Majority neighborhood number, Complement connected Majority neighborhood.*

1. Introduction

In this article the author use the simple and undirected graph. The parameters $n_0(G), n_c(G)$ and $n_{CM}(G)$ was studied in the following articles [1],[5],[6]-[8].

2. Majority neighborhood connected complement number of a graph

Definition 2.1

Let G be a graph with m vertices and n edges. A complement connected majority neighborhood set S_{CMN} such that S_{CMN} is a connected majority neighborhood set in G and majority neighborhood set in$\bar{G}$. The complement majority neighborhood number is the minimum size of connected complement majority neighborhood set of G and is denoted by $n_{\overline{ccm}}(G)$.

Theorem

If $G = C_r$ with $r \geq 4$ then $n_{\overline{CCM}}(G) = \left\lceil\frac{r-3}{2}\right\rceil$.

Proof

If $G = C_r, |V(G)| = r = |E(G)|$, the set $S_{cM}(G) = \{v_1, v_2, v_3, \ldots v_{\left\lceil\frac{r-3}{2}\right\rceil}\}$ be the CMN- set . In $\bar{G}S_{cM}(G)$ covers $\left\lceil\frac{r-3}{2}\right\rceil$ vertices and $\deg(v_i) = r - 3 \in \bar{G}$ and$|E(\bar{G})| = r(r-3)$, each$|\langle u_i\rangle| = r(r-3) \geq \frac{r(r-3)}{2} \in \bar{G}$ and each $N[u_i] \geq \left\lceil\frac{r-3}{2}\right\rceil$. Hence $n_{\overline{CCM}}(G) = \left\lceil\frac{r-3}{2}\right\rceil$.

Theorem

For the path graph $P_n, n_{\overline{ccm}}(G) = \left\lceil\frac{p-3}{2}\right\rceil, p \geq 4$.

Theorem

For the $G = F_{2r+1}$ with $r \geq 1$ then $n_{\overline{ccm}}(G) = 2$

Proof: Let $G = F_{2r+1}$ with $|V(G)| = 2r + 1$ and $E(G) = 3r$in G. Let $S_{cM}(G) = \{v, u_1\}$ be the connected neighborhood setin G. Since $|n(v)| = 2r + 1, |\langle n(v)\rangle| = 3r$ and by the super hereditary property $n(G) \geq n_M(G)$. Therefore $S_{cM}(G)$ is connected majority neighborhood set. In complement of G, $|V(\bar{G})| = 2r + 1$and $|E(\overline{G})| = r(2r-2)$. $d(v) = 0, d(u_i) = 2r - 2 \in \bar{G}$. Therefore, $|n[u_i]| = 2r - 1, u_i \in S_{cM}(G)$and $|\langle n[u_i]\rangle| = r - 2(2r-2) \Rightarrow |n[u_i]| = 2r - 1 \geq \left\lceil\frac{2r+1}{2}\right\rceil = \left\lceil\frac{p}{2}\right\rceil \in \bar{G}$, $|\langle n[u_i]\rangle| = r - 2(2r-2) \geq \left\lceil\frac{r(2r-2)}{2}\right\rceil = \left\lceil\frac{q}{2}\right\rceil \in \bar{G}$. Therefore, $S_{cM}(G)$ is majority neighborhood set.Hence $n_{\overline{ccm}}(G) = 2$.

Theorem

If $G = K_{1,r}$ with $r \geq 2$then $n_{\overline{ccm}}(G) = 2$.

Theorem

For the wheel graph $W_r$with $r \geq 3 n_{\overline{ccm}}(G) = \begin{cases} 2 & for\ r = 3,4\ r \geq 8 \\ 3 & for\ \ \ \ r = 5,6,7 \end{cases}$

Proof. Let $G = W_r$ with $r \geq 3$ and$|V(G)| = r, |E(G)| = 2r$.

Case (i):$r = 5,6,7$
If $G = W_5$then $S_{cM}(G) = \{v, u_1, u_2\}$, $d(v) = \Delta(G)$ and $d(u_1) = d(u_2) = 3$. $|V(G)| = r \ and \ |E(G)| = \left\lceil \frac{r}{2} \right\rceil - 1 = q \in \bar{G}$. Therefore $\langle S_{cM}(G) \rangle$covers 2 edgesin$\bar{G}$. $|\langle n[u_1, u_2] \rangle| = 2 = \left\lceil \frac{r}{2} \right\rceil - 1 \geq \left\lceil \frac{q}{2} \right\rceil \in \bar{G}$. Hence $S_{cM}(G)$ is $n_{\overline{ccm}}$ set.

Case (ii) $r \geq 8$, If $G = W_r, r \geq 8$ then Let $S_{cM}(G)$ be the connected neighborhood set in G, $S_{cM}(G) = \{v, u_1\}$, . $|V(G)| = r$and$|E(G)| = 2(r-1)$ and $\deg(v) = \Delta(G). \{v\} \subseteq S_{cM}(G)$, therefore $S_{cM}(G)$ is a connected majority neighborhood set.
In $\bar{G}$, $|E(\bar{G})| = \left\lceil \frac{r}{2} \right\rceil \times \deg(u_i) - \frac{\deg(u_i)}{2}$, and each vertex ofdeg$(u_i) = r - 3 \geq \left\lceil \frac{r}{2} \right\rceil$ and $\deg(u_i) = |E(\bar{G})| = \left\lceil \frac{r}{2} \right\rceil \times \deg(u_i) - \frac{\deg(u_i)}{2} = \deg(u_i) \geq \left\lceil \frac{|E(\bar{G})|}{2} \right\rceil$.
Hence, $n_{\overline{CCM}}(G) = 2$.

Theorem

For $G = D_{r,s}$, with $r, s \geq 1$ then $n_{\overline{CCM}}(G) = 2$
Proof: Let G be a graph with $V(G) = \{X_1 \cup X_2\}$,$X_1 = \{v, v_1, v_2 \dots v_r\}$, $X_2 = \{u, u_1, u_2, u_3, \dots u_s\}$. The $\Delta(G) = u$ and $\delta(G) = v$.Therefore$|\langle u \rangle| = \Delta(G) \geq \left\lceil \frac{q}{2} \right\rceil \geq \left\lceil \frac{p}{2} \right\rceil$.Hence $n_{CCM}(G) = 1 \in G$. But in$\bar{G}$, $\langle u \rangle$covers $\{X_1 - v\} \leq \delta(G) \leq \left\lceil \frac{q}{2} \right\rceil$, therefore it is not a majority neighborhood set. Let Choose the set $S_{cM}(G) = \{u, v\}$, $|\langle u, v \rangle| = \Delta(G) + \delta(G) \geq \left\lceil \frac{q}{2} \right\rceil \geq \left\lceil \frac{p}{2} \right\rceil$. Hence $n_{\overline{CCM}}(G)$.

Theorem

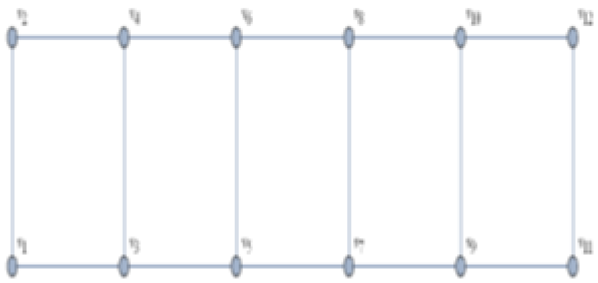
If $G = P_2 \times P_t$ with $t \geq 3$ then $n_{\overline{CCM}}(G) = \left\lceil \frac{3t-1}{4} \right\rceil$.

Theorem

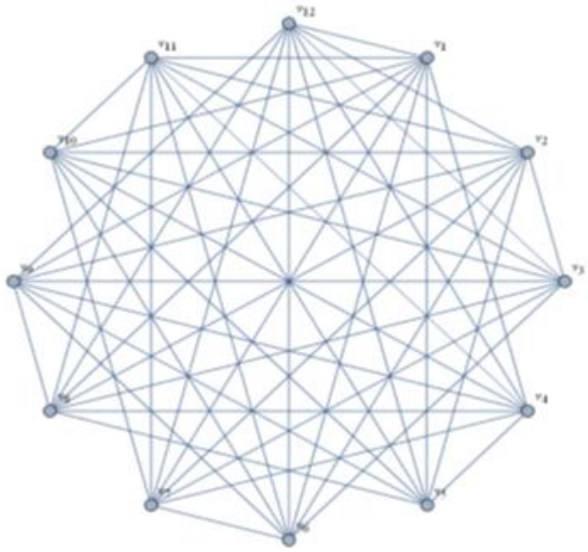
If $G = P_3 \times P_t$ with $t \geq 4$ then$n_{\overline{CCM}}(G) = \left\lceil \frac{5(t-1)}{6} \right\rceil$

Theorem

If $G = P_4 \times P_t$ with $t \geq 5$ then $n_{\overline{CCM}}(G) = \left\lceil \frac{5(t-1)+1}{4} \right\rceil$
Example



$G = P_2 \times P_6$

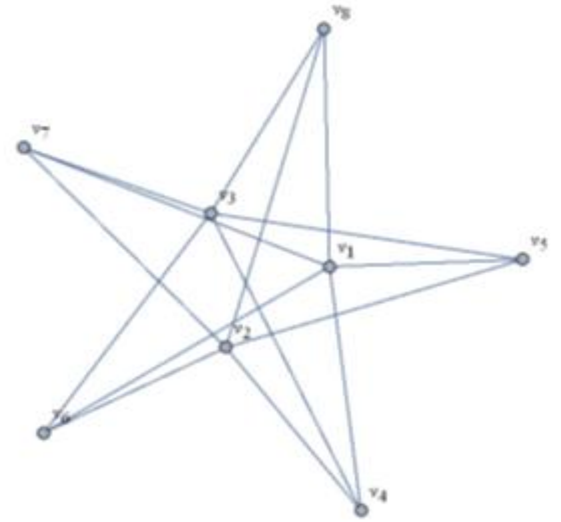


$\bar{G} = \overline{P_2 \times P_6}$

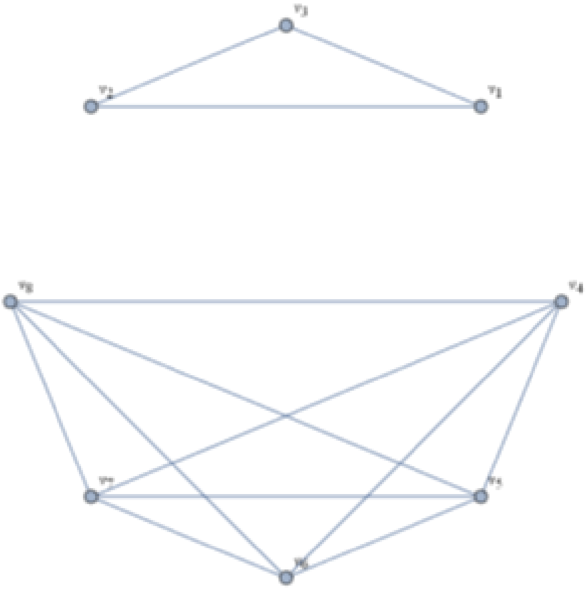
Theorem

For $G = K_{r,s}$ with $r \leq s$, $n_{\overline{CCM}}(G) = 2$

Proof
Let $G = K_{r,s}$ with$\{X_1 \cup X_2\} = V(G)$, $|V(G)| = r + s$. Let $S_{cM}(G) = \{u_i\}, u_i \in X_1$ be the CMN-Set in G but not in$\bar{G}$. Since $\bar{G} = K_r \cup K_s$ therefore$|\langle n[u_i] \rangle| = \frac{r(r-1)}{2}$and $|\langle n[u_i] \rangle| = \frac{s(s-1)}{2}$, $|\langle n[u_i] \rangle| = \frac{r(r-1)}{2} < |\langle n[u_i] \rangle| = \frac{s(s-1)}{2}$. Hence, $S_{cM}(G) = \{u_i\}$, $u_i \in X_1$not a CMN-set. Suppose$S_{cM}(G) = \{v_i\}, v_i \in X_2$ be the CMN-set in$\bar{G}$ but not in G.
Therefore $S_{cM}(G) = \{u_i, v_i\}, u_i \in X_1 \& v_i \in X_2$,
$|\langle S_{cM}(G) \rangle| = \frac{s(s-1)}{2} + \frac{r(r-1)}{2} \geq \left\lceil \frac{E(\bar{G})}{2} \right\rceil$and $|N[u_i, v_i]| \geq \left\lceil \frac{V(\bar{G})}{2} \right\rceil$. Hence $n_{\overline{CCM}}(G) = 2$



$G = K_{3,5}$



$\bar{G} = K_3 + K_5$

$n_0(G) = 2, n_m(G) = n_{CM}(G) = 1$, $n_{\overline{CCM}}(G) = 2$

Observation

In some graph G the number (i) $n_0(G) = n_{\overline{CCM}}(G) = 2$
(ii) $n_{CM}(G) \leq n_{\overline{CCM}}(G)$

3. Conclusion

The author introduced the new parameter $n_{\overline{CCM}}(G)$ in this article. Further the we find this number for various classes of graphs and bounds.

References

[1] FrankHarary, Graph theory, Addision-Wesley reading MA(1969).

[2] Haynes,T.WHedetniemi,S.T., and Slater,P.J, Fundementals of domination in graphs, Marcel Dekker. Inc., New York, (1998).

[3] Paulraj Jayasimman and Joseline Manora, Connected Majority Neighborhood Number of a Graph, International Journal of Research, Vol. 7, 6, 2018, PP-984-987.

[4] Sampathkumar.E and Prabha S. Neeralagi, Neighborhood number of a graph, Indian J. Pure.Appl.Math.,16(2) 126-132.Feb.1985

[5] Joseline Manora .J and Swaminathan .V, Majority neighborhood number of a graph- published in Scientia Magna, Dept. of Mathematics, Northwest Universtiy, Xitan, P.R China – Vol (6), N0.2, 20-25(2010).

[6] Renuka, Balaganesan, “Felicitous labelling of some path related of graphs”, Indian Journal of public Health Research & Development, Vol.9, Sep-2018.

[7] M. Mehendran, Nithyaraj, Balaganesan, “The split Monophonic Number of a Graph”, Journal of advance research in Dynamical & control systems, Vol.11, 01, Specl.Issue-2019

[8] Venkatesan, Balaganesan, Vimala, “Fuzzy Matrix with Application in Decision Making”, International Journal of pure and Applied Mathematics, Vol.116, No.23, 2017, PP.551-554.